

HW6:

1. Using Proposition 10.3.2 we define the following sets:  $X$  = the set of students who liked Reasoning,

$Y$  = the set of students who liked Algebra,  $Z$  = the set of students who liked Calculus. We want to apply the formula

$$|X \cup Y \cup Z| = |X| + |Y| + |Z| - |X \cap Y| - |X \cap Z| - |Y \cap Z| + |X \cap Y \cap Z| \quad (1)$$

to compute the total of students that like at least one of the courses. The answer we are looking for is

then  $182 - |X \cap Y \cap Z|$

So we compute the cardinalities of the sets in the right-hand side of the equation.

$$|X| = 129$$

$$|Y| = 129$$

$$|Z| = 129$$

$$|X \cap Y| = 85$$

$$|X \cap Z| = 86$$

$$|Y \cap Z| = 89$$

$$|X \cap Y \cap Z| = 54$$

And formula (1) yields:

$$|X \cap Y \cap Z| = 129 + 129 + 129 - 85 - 86 - 89 + 54 = 181$$

So our final answer is  $182 - 181 = 1$  student.

3. We proceed to prove (iii), (i), and (ii).

(iii) Again by using Proposition 10.3.2, let So we compute the cardinalities of the sets in the righthand

side of the equation.

$x$  = the number of those who speak only English.

$y$  = the number of those who speak only Spanish.

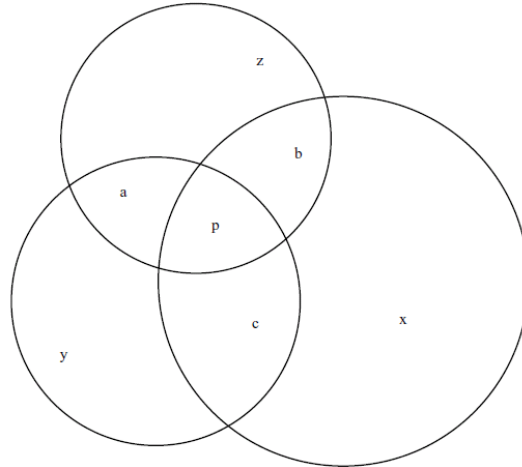
$z$  = the number of those who speak only Swahili.

$a$  = the # of those who speak only Sp. & Sw.

$b$  = the # of those who speak only En. & Sw.

$c$  = the # of those who speak only Sp. & En.

$p$  = the # of those who speak all three.  $y$



This gives the following equation,

$$\begin{aligned}
 100 &= 75 + 60 + 45 - a - p - b - p - c - p + p \\
 &= 180 - p - (a + b + c + p) \quad (2)
 \end{aligned}$$

Notice that  $100 - (a + b + c + p)$  is precisely the number  $I$  of individuals that speak only one language, hence we get

$$I = 20 + p \quad (3)$$

with  $p = 40 - 1$

$2n, n = a + b + c$ . Equation (3) give the required relation.

(i) By (iii) the formula relating the number of individuals speaking only one language  $I$  is related to the number of individuals that speak all three  $p$  by the formula  $I = p + 20$ . Also by (iii) the maximum value for  $p$  is 40 and it also implies that in this case no individual speaks exactly two languages. Hence the max is 60 and there are 35,20, and 5 speakers of English, Spanish, and Swahili respectively.

(ii) Consider the English speaking people. Exactlyyy, 25 can't speak English. For having the most number of solely-En. We can have 25 out of 60 of Sp. speaking, and 25 out of 45 Sw speaking outside En. So, even if Sw is contained in Sp, we have 35 in common with En.

Therefore, we have at most 40 only-En.-speaking.

In this case, we had Sw contained in Sp. That is no one speaks only Sw.

For Sp in this case, depending on the common members with En, it can have 15 at most, and 0 only-Sp speaking.