HW-4:

P . 117::

11-

(i) True,let y=1-x. Then the inequality holds.
(ii) True, let y=x-1. Then the inequality holds.
(iii) False, because for each x, we can have y=-x-1, which results in x+y=-1<0.
(iv) False, for x=0 there does not exist such y.
(v) For each x, let y=-x, then xy=-xx<0.
(vi) True. Let y be always 0!
(vii) True, the statement holds for x=0.
(viii) True, let y=-x, then we have y+x=0.
(ix) False, since a number cannot be both greater and equal zero.
(x) True, as both statements are true statements.

12-

Let A={1,2,3}. Obviously, it is a subset of integers, and has 3 as the greatest number.

Also let A={1,3,5,7,….}, that does not have any largest element!

$$\exists N \in A \subseteq \mathbb{Z} : \; y \in A \Longrightarrow y \leq \; N$$

13-

$$A \times (B \cup C) = \{(x, y)|x \in A \; and \; ((y \in B)or(y \in C))$$

$$(A \times B) \cup (A \times C) = \{(x, y\}|\big((x \in A)and(y \in B)\big)OR \; \big((x \in A)and(y \in C)\big)\}$$

By expanding the statement within $\{\dots\}$, that is using $(p \wedge q) \vee (p \wedge r) = p \wedge (r \vee q)$ we find out that the two sets are equal.

(ii)

$$(A \times B) \cap (C \times D) = \{(x, y\}|\big((x \in A)and(y \in B)\big)AND \; \big((x \in C)and(y \in D)\big)\}$$

But as $(p \wedge q) \wedge (r \wedge s) = (p \wedge r) \wedge (q \wedge s)$ we have

$$(A \times B) \cap (C \times D) = \{(x, y\}|\big((x \in A)and(x \in C)\big)AND \; \big((y \in B)and(y \in D)\big)\}$$

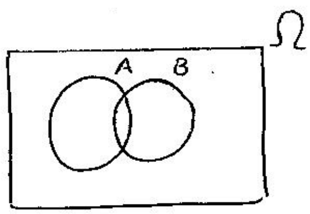
Which is exactly $(A \cap C) \times (B \cap D)$.

15-(i) $\chi_A(x)\chi_B(x) = 1 \iff (\chi_A(x) = 1 \text{ and } \chi_B(x) = 1) \iff (x \in A \text{ and } x \in B) \iff x \in A \cap B$

$$\chi_A(x)\chi_B(x) = 0 \iff (\chi_A(x) = 0 \text{ or } \chi_B(x) = 0) \iff (x \notin A \text{ or } x \notin B) \iff \sim(x \in A \text{ and } x \in B)$$
$$\iff \sim(x \in A \cap B) \iff x \notin A \cap B$$

We made use of the facts that 1)Chi can be either 0 or 1, 2)the product of two real numbers is zero iff at least one of them is zero.

(ii)



By an easy computation we find out that:

	χ_A	χ_B	χ_C	$\chi_{A\cup B}$
$x \in A \wedge x \in B$	1	1	1	1
$x \in A \wedge x \notin B$	1	0	1	1
$x \notin A \wedge x \in B$	0	1	1	1
$x \notin A \wedge x \notin B$	0	0	0	0