HW2:

#Prove that 2n+1 is an odd number.

Assume not, that is it is an even number. An even number can be divided by 2, in other words, it is of the form 2k for some integer k.

Assume $2n + 1 = 2k$ for some integer k.

By taking $2k$ to the other side, we get $2(n - k) = 1$. But this implies that $2 \leq 1$, which is impossible.

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7- Assume the statement is false, that is we assume that n is not even, and show that n^2 is not even. Logically, this is equivalent to n^2 *is even* $\rightarrow n$ *is even*.

We know that every non-even number is of the form 2k+1 for some $k \in Z$. Therefore: $n^2 = (2k+1)^2 = 4k^2 + 4k + 1$. We, now, show that this expression is always an odd number. $n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 = 2\bar{k} + 1$ and obviously, $\bar{k} \in Z$.

Hence, n^2 is odd.

To add up, we proved that $n \text{ is } \textbf{not} \text{ even} \rightarrow n^2 \text{ is } \textbf{not} \text{ even}$, which is equivalent to $n^2 \text{ is even} \rightarrow n \text{ is even}$.

8- (i) $x^2 - x - 2 = (x+1)(x-2) = 0$. We know that for the product of two real numbers to be zero, we must have, at least, one of the two real numbers equal to zero. That is, in our case,

$\big((x+1) = 0\big) \vee ((x-2) = 0)$.

The other side, that is $\Leftarrow$ is obtained by sustitution.

(ii) $x^2 - x - 2 = (x+1)(x-2) > 0$ we know that for the product ab of real numbers is positive $\Leftrightarrow (a > 0 \text{ and } b > 0) \vee (a < 0 \text{ and } b < 0)$. In this problem, we have $\Leftrightarrow (x > 2 \text{ and } x > -1) \vee (x < 2 \text{ and } x < -1)$. Evidently, $(x > 2 \text{ and } x > -1)$ means x>2. And $(x < 2 \text{ and } x < -1)$ means

X<-1. This completes the proof, by noting that we can return on the same path!

9- Assume that we have a number N as the largest number. But we know that every natural number has got a successor, $N + 1$. As N is the largest integer, $N + 1 \leq N$. As we can add and subtract to both sides of this inequality, we can subtract N. So we`ve got $1 \leq 0$, which we know is wrong, that is not $0 = 1$ nor $0 > 1$.