

HW10;

4.3; We want to show that the point "between" A & C, that is \overline{AC} , lie inside $\angle ABD$.

We know that $A \in \overrightarrow{AC}$ and $A \in \overrightarrow{AD}$. To prove that an arbitrary point $P \in \overrightarrow{AC}$ lies inside the angle, we assume the negation of the statement: Suppose $\exists P \in \overrightarrow{AC}$: P is not inside the angle, that means the line \overline{PC} crosses ~~either~~ either \overrightarrow{AD} or \overrightarrow{AB} .

(By definition of point inside). So, ~~if~~ there is $Q \in \overline{PC}$ s.t.

$Q \in \overrightarrow{AD}$ or $Q \in \overrightarrow{AB}$. Recall that $Q \in \overline{PC} \subseteq \overline{AC} \subseteq \overrightarrow{AC}$.
By incidence axiom, as $a \in \overrightarrow{AD}, \overrightarrow{AB}$ and $\begin{cases} Q \in \overrightarrow{AB} \\ Q \in \overrightarrow{AD} \end{cases}$ OR,

the line $\overrightarrow{CA} = \overrightarrow{QC} = \overrightarrow{PC}$ must coincide with

either \overrightarrow{AB} or \overrightarrow{AD} ; which contradicts 'C' being ~~inside~~ inside $\angle DAB$.

4.4; By paragraph '3' of protractor axiom, \exists unique ray: $\overrightarrow{AD} \subseteq \mathcal{H}$,

such that $m\angle BAC = \alpha$. By exercise 3.9, the point C is specified unique on \overrightarrow{AD} .

exercise 3.12; As we proved before, we can construct a coordinate system s.t. $f(A) = 0$. As $f(\cdot)$ is a correspondence between \mathbb{R} and \overrightarrow{AD} (not the ray, the whole line), there are $P \neq Q \in \overrightarrow{AD}$ such that

$f(P) = r$, $f(Q) = -r$. If P & Q both lie outside the ray \overrightarrow{AD} ,

it is a contradiction, For $f(A) = 0$, $-r < f(A) < r$, ~~with~~. So

at least one of them lies in \overrightarrow{AD} , say P. $\rightarrow |f(P)| = |f(P) - f(A)| =$

$$|PA| = r. \quad \square$$

4.7; By exercise 4.2, \overline{BC} lies inside the angle. So if $\overline{BC} \cap \overrightarrow{AD} \neq \emptyset \rightarrow$
 \overrightarrow{AD} entirely lies in the angle. ~~If \overrightarrow{AD} had to have a point~~
~~outside the angle, Because all points of \overrightarrow{AD} are on the same~~
side of both both \overrightarrow{AB} and \overrightarrow{AC}