

1. (4 pts) Rewrite “It is not the case that the car is both red and has leather upholstery” as an equivalent sentence that uses ‘or’, but not ‘and’.
2. (4 pts) Rewrite without negatives (without using “not”):
not $(\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \exists z \in \mathbb{R}, x < y \Rightarrow x < z < y)$.
3. (6 pts) Prove that

$$[(A \cup B) - (A \cap B)] \cap C = [(A \cap C) \cup (B \cap C)] - (A \cap B \cap C)$$

4. (8 pts) Prove that

$$\sum_{j=1}^n j^3 = \left(\sum_{j=1}^n j \right)^2.$$

You may assume that

$$\sum_{j=1}^n j = \frac{n(n+1)}{2}.$$

5. True or False (Prove your answer or give a counterexample)
 - (a) (3 pts) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, 0 < x + y < 1$.
 - (b) (3 pts) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \forall z \in \mathbb{R}, z < y \Rightarrow xz < 0$.
 - (c) (3 pts) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, y > 0 \Rightarrow y - x > 1$.
 - (d) (4 pts) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, -x < y < x \Rightarrow y^2 < 0.1$.
 - (e) (4 pts) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x \neq 1 \Rightarrow xy = x + y$.
 - (f) (6 pts) $(B \cap C) - A \subseteq C - (A \cap B)$.

6. (5 pts) Show tht the sequence

$$a_n = \frac{1}{2n+3}$$

is null, i.e. $\forall \varepsilon \in \mathbb{R}^+, \exists N \in \mathbb{Z}, n \geq N \Rightarrow |a_n| < \varepsilon$.

7. (8 pts) Let $g : X \rightarrow Y$ and $f : Y \rightarrow Z$. Show that if $f \circ g$ is a bijection then g is an injection and f is a surjection.