MAT 260 Problem Solving in Mathematics
Week 2 : Combinatorics

1. (i) I have twelve books. In how many ways can I line them up on a single shelf?
(ii) In how many ways can I choose seven of them and line them up on a single shelf?
(iii) In how many ways can I choose seven to take to school?

2. (i) I have to choose a password consisting of exactly six letters (no numbers, symbols, etc). How many choices are there, if the letters can be repeated? [Hint: There are 26 letters in the alphabet!]

(ii) Somebody hacked into my account, so I need to choose a better password. This time I will choose seven letters, but they cannot be repeated. How many choices are there? (Is this more secure?)

(iii) Somebody hacked into my account again! This time my password will be six characters, which can be letters or numbers, and there must be at least one letter and one number. How many choices are there? (Is this more secure?)

3. (i) An ice-cream vendor sells eleven flavours of ice-cream. In how many different ways can I buy six cones, some or even all of which could be the same flavour?

(ii) An ice-cream vendor sells six flavours of ice-cream. In how many different ways can I buy eleven cones, some or even all of which could be the same flavour?

4. There are 33 boys, and they want to divide into three teams of eleven. In how many different ways can this be done?

5. Choose n distinct points on a circle and draw all possible chords. How many points of intersection are there inside the circle? Does it depend on how the n points are chosen?

6. (i) I want to bring twelve sticks of chalk to my class. There are five colours, and I want to bring at least one stick of each colour. How many different combinations are there?

(ii) I decided I'd better use white chalk too, in addition to the five colours. So I'll bring fifteen sticks altogether, at least one of each colour, and at least five of which are white. How many different combinations are there?

7*. A group of five professors are setting a mathematics competition. When they go home at night, they leave their work in a room which has a certain number of locks on the door. Each professor has keys to some, but not all of the locks. In fact, any three professors will have enough keys between them to open the door, but any two professors will not have enough. What is the smallest number of locks needed, and how many keys will each professor have?