MAT 260 Problem Solving in Mathematics
Week 10 : Geometry

1. Find the areas of the following octagons, which are inscribed in circles.

2. (i)* Let the chords from A to B and from C to D intersect at the point P. Prove that

$$(AP)(PB) = (CP)(PD)$$

where AP denotes the length of the segment from A to P, and likewise for the other segments.

(ii) Let P lie anywhere inside the angle $\angle AOB$, and draw a line through P meeting the edges of the angle at X and Y. Show that the quantity

$$(XP)(PY)$$

is minimized when $OX = OY$.

[Hint: Draw a circle which is tangent to the angle at X and Y, then use part (i) to show $(XP)(PY) < (X_2P)(PY_2)$.]

3. In the equilateral triangle $\triangle ABC$, the distances from an interior point P to the vertices is 5, 6, and 7. Find the length of the edges of the triangle.

4. Choose a vertical segment CD in a semicircle, and inscribe a circle in the part of the semicircle to the right of CD, as shown. If the circle touches AB at the point E, prove that $\triangle ADE$ is isosceles, i.e. prove that the segments AD and AE have the same length. [Hint: Use the Pythagorean Theorem. Lots.]

5. A straight line cuts the hyperbola $y = x^{-1}$ at the points P and Q, and cuts the x and y-axes at A and B, as shown. Prove that the segments AP and BQ have the same length.

6. Suppose that the line $x/a + y/b = 1$ is tangent to the circle $x^2 + y^2 = c^2$. Find a relation between a, b, and c.