MAT 320. HW 10, now due Nov 15, 2018

Do problems 21.5, 21.6, 21.9, 21.10, 21.11 from the textbook.

Problem 1. Let S^1 be the unit circle $S^1 := \{x, y : x^2 + y^2 = 1\} \subset \mathbb{R}^2$. Consider the map $f : \mathbb{R}^2 \setminus \mathbf{0} \to S^1$, obtained by projecting from the origin $\mathbf{0} \in \mathbb{R}^2$. Write out f in coordinates and prove that f is continuous.

Problem 2. Think of $\mathbb{R} \subset \mathbb{R}^2$ as the x-axis. Let then f be the projection from the point $(0, 1) \in \mathbb{R}^2$, which maps $S^1 \setminus \{(0, 1)\}$ onto $\mathbb{R}$. Prove that f is continuous, and let then $\tilde{f} : S^1 \to \mathbb{R} \cup \{\infty\}$ be defined to be the extension of f obtained by setting $\tilde{f}(0, 1) := \infty$. Describe the notion of topology on $\mathbb{R} \cup \{\infty\}$ that makes $\tilde{f}$ and $\tilde{f}^{-1}$ continuous maps. Does the map $x \mapsto 1/x$ then extend to a continuous map $g : \mathbb{R} \cup \{\infty\} \to \mathbb{R} \cup \{\infty\}$?