

MAT 203 MIDTERM (SPRING 2006)

Instructions: Solve each of the following five problems on a separate sheet of paper. Use a separate sheet of paper for each problem, and clearly label each problem. Show the important steps of your work, as well as the final answer. You will receive no credit for answers which are not backed up by work shown or by an explanation given.

- (1) Set $\mathbf{v} = \mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$ and $\mathbf{w} = -3\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$, then answer the following.
- (a) Is \mathbf{v} orthogonal (i.e. perpendicular) to \mathbf{w} ?
 - (b) Compute $3\mathbf{v} - 2\mathbf{w}$?
 - (c) Find a non-zero vector which is orthogonal (i.e. perpendicular) to both \mathbf{v} and \mathbf{w} .
 - (d) Compute the length (i.e. the norm) of \mathbf{v} .
- (2) Find an equation for the plane in 3-space which contains the three points $(0,1,1)$, $(2,1,-1)$, $(1,2,3)$.

- (3) A particle is moving in 3-space. Its acceleration at time t is the vector valued function

$$\mathbf{a}(t) = -\cos(t)\mathbf{i} + e^t\mathbf{j} + 6t\mathbf{k}.$$

Find a formula for its velocity at time t (denoted by $\mathbf{v}(t)$) and its position at time t (denoted by $\mathbf{r}(t)$) if its velocity and position at time $t = 0$ are given by

$$\mathbf{v}(0) = \mathbf{i} + \mathbf{j} - \mathbf{k}$$

and

$$\mathbf{r}(0) = 2\mathbf{i}.$$

- (4) If $f(x, y) = \sin(x + y) - x^2y$ then compute the following.
- (a) $f_x(x, y) = ?$
 - (b) $f_y(x, y) = ?$
 - (c) $\nabla f(x, y) = ?$
 - (c) $f_{xyx}(x, y) = ?$

- (5) Suppose that $f(x, y)$ satisfies $f_x(0, 1) = 2$ and $f_y(0, 1) = -3$. Then answer the following.

- (a) If $g(s, t) = f(st^2 + t, e^{st})$, then what is the value of the partial derivative $g_t(2, 0)$?
- (b) Find the unit vector \mathbf{u} for which the directional derivative $D_{\mathbf{u}}f(0, 1)$ is maximal.