Math 360 (Spring '16)

Homework 9

due on May 5

1. Let ℓ be a line, and $P \notin \ell$ a point.

   i) What is the locus of points at fixed distance x from P?

   ii) What is the locus of points at fixed distance y from ℓ?

   iii) Find a point Q which is at distance x from P and distance y from ℓ.

   iv) How many points Q are at distance x from P and distance y from ℓ? (N.B. here you should get different answers depending on the __________ between ℓ and P)

2. Given a segment AB and a point M on this segment:

   i) Construct a point P such that $\angle APB = 60°$. What is the locus of points P with this property?

   ii) Construct a point P such that $\angle APB = 60°$ and PM is the bisector of angle $\angle P$.

3. (This exercise tests the use of sine/cosine laws)

   1) Compute $\sin 60°$ and $\cos 60°$ (Hint: use an equilateral triangle).

   2) Given a triangle ABC such that $\angle BAC = 60°$, $AB = 2$, $AC = 5$, compute BC and then the other two angles (i.e. sin or cos of those angles).

   3) Decide if the angles at B and C are acute or obtuse. (Before you do any computation, which angle could be obtuse - justify)

   4) Compute the distance from A to the line BC.

4. You are given segments of length $a, b, c, \ldots$ and if needed a segment of length 1. Construct the following quantities and indicate if you need to use the unit segment.

   i) $a\sqrt{2}$

ii) $\sqrt{2a}$

iii) $\frac{a^2c}{b^2}$

iv) $\frac{1}{a}+\frac{1}{b}$

v) $\sqrt{a^2+bc}$

5. Let $T(\vec{x}) = A\vec{x} + \vec{b}$ be an affine transformation.

   i) Give an example of affine transformation such that

   $$T\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

   ii) List all affine transformations that preserve the origin and the y-axis.

   iii) Prove that an affine transformation that preserves both the x-axis and y-axis, preserves also the origin. List all such transformations.

   iv) Find an affine transformation T that transforms the triangle with vertices $A = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $B = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ into the standard triangle (vertices $\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$).

6. Prove using affine geometry that the medians in a triangle meet in a single point.