Math 360 (Spring '16)

Homework 8

due on Apr 26

1. Determine which of the following transformations $t : \mathbb{R}^2 \to \mathbb{R}^2$ are Euclidean transformations. How about affine transformations?

   a) $t(x) = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} x + \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

   b) $t(x) = \begin{pmatrix} -\frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} x + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

   c) $t(x) = \begin{pmatrix} -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & -\frac{4}{\sqrt{5}} \end{pmatrix} x + \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

2. Write down an example (if one exists) of each type of transformation $t : \mathbb{R}^2 \to \mathbb{R}^2$ described below. In each case, justify your answer.

   (a) An affine transformation t which is not a Euclidean transformation.

   (b) A Euclidean transformation t which is not an affine transformation.

   (c) A transformation t which is both Euclidean and affine.

   (d) A transformation t which is one-to-one, but is neither Euclidean nor affine.

3. Which of the following are affine properties (i.e. preserved by affine transformations)?

   – distance

   – collinearity

   – circularity

   – magnitude of angle

– midpoint of line segment

4. The affine transformation $t : \mathbb{R}^2 \to \mathbb{R}^2$ is given by

$$t(x) = \begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix} x + \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

Determine the image under t of each of the following geometric objects:

a) $2y = 3x - 1$

b) $x^2 + y^2 = 1$.

5. Determine the affine transformation which maps the points $(1, -1)$, $(5, -4)$ and $(-2, 1)$ to the points $(1, 1)$, $(4, 0)$ and $(0, 2)$ respectively.