

MAT 132 Practice Mid Term

Name:

ID Number:

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. No work, no credit.
- Calculators are not permitted.
- Write clearly. If the grader is unable to understand what you have written, you may receive no credit.
- Attempt all problems. There are eight questions, few with more than one parts. Maximum possible score is 130.
- Good luck!

1. (10 points) Using the definition of definite integral as the limit of a sum, evaluate the following (do not use the fundamental theorem of calculus!)

$$\int_1^2 (x^2 + 2x) dx$$

Hint: $-(a + b)^2 = a^2 + 2ab + b^2$, $\sum_{i=1}^n i = \frac{n(n+1)}{2}$, $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$

2. (25 points) Evaluate the following:-

a. (5 pts) Given that $\int \ln x dx = x \ln x - x$, evaluate

$$\int \ln(4x + 7) dx$$

b. (10 pts) Evaluate $\int x^n \ln(x) dx$, in terms of n .

Hint:- Use integration by parts.

c. (10 pts) Evaluate $\int x^2 \ln(x-1) dx$.

Hint:- Make a substitution. Part (b) might also be useful.

3. (20 points)

a. (10 pts) Evaluate

$$\int_0^1 (x^3 + 3x^2)\sqrt{x^4 + 4x^3} dx$$

b. (10 pts) Evaluate

$$\int \frac{1}{x(x^2 + 1)} dx$$

4. (20 points) Determine whether the following integrals are convergent or divergent.
a. (10 pts)

$$\int_1^{\infty} \frac{1}{1+x^2} dx.$$

In this problem, if the integral is convergent, then find the actual value.

b. (10 pts)

$$\int_1^{\infty} \frac{\cos^4(1+x^2)}{1+3x^2+4x^8} dx$$

5. (10 points) Find the area of the region bounded by the curves $x = 5y - y^2$ and $y = x$. Please note that if you do not draw a correct picture, you automatically get zero for this problem!

6. (*10 points*) The base of a solid is an equilateral triangle of side s . Parallel cross-sections perpendicular to the base are semicircles. Find the volume of the solid. Again, if you do not draw a correct picture, you get a zero!

7. (10 points) Let S be the solid formed by rotating the region bounded by the curves $x = \frac{y^2}{4}$ and $x = 5 - y^2$ about the y -axis. Find the volume of S . Again, you must draw a correct picture!

8. (25 points) **a.** (5 pts) Set up, but do not evaluate, an integral that represents the length of the curve,

$$y = \sin(2x), \quad 0 \leq x \leq 2\pi$$

b. (5 pts) Find the average value of the function $f(x) = \frac{x^3+1}{1+x^2}$, from -1 to 1 .

c. (10 pts) A chain lying on the ground is $10m$ long and its mass is $80kg$. How much work is required to raise one end of the chain to a height of $6m$?

d. (5 pts) Find the approximate value of $\int_1^5 (1+x^2)dx$, using left end point approximation and using $n = 4$.