

Applied Algebra, MAT312/AMS351
Practice Problems for the Final

- (1) Find the greatest common divisor of $12n + 1$ and $30n + 2$.
- (2) Prove that the product of three consecutive natural numbers is always divisible by 6.
- (3) Solve the following linear congruences
 - (a) $26x \equiv 8 \pmod{44}$;
 - (b) $24x \equiv 9 \pmod{40}$.
- (4) Solve the following system of linear congruences:

$$\begin{cases} x \equiv 4 \pmod{25} \\ 3x \equiv 6 \pmod{39} \end{cases}$$

- (5) Show that the equation $5x^7 - x^4 = 23$ has no integer solutions.
- (6) Recall that the Fibonacci sequence is defined as $F_1 = 1, F_2 = 1$, and then for every $n > 2$, $F_n = F_{n-1} + F_{n-2}$. Prove that for every n , $F_2 + F_4 + \cdots + F_{2n} = F_{2n+1} - 1$.
- (7) Find the last two digits of the number 3333^{4444} .
- (8) Let G be a group and $C = \{a \in G : ax = xa \text{ for all } x \in G\}$. Prove that C is a subgroup of G .
- (9) Let R be a relation on \mathbb{Q}^\times (nonzero rational numbers) defined by:

aRb if and only if ab is a square of a rational number.

Prove that R is an equivalence relation.

- (10) Let $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 1 & 6 & 7 & 4 & 2 \end{pmatrix}$.
 - (a) Compute $(145)\pi$.
 - (b) Determine the order of π .
 - (c) Determine the sign of π .
- (11)
 - (a) What is the order of the group $S(4)$?
 - (b) What are the possible orders of elements in the group of order 24?
 - (c) What are the possible orders of permutations in the group $S(4)$?
- (12) Let a, b, c be elements of some group G . Solve the equation $(ax)(bc) = e$ in G . Justify every step.
- (13)
 - (a) Let H be the subgroup of G_{15} generated by $[4]_{15}$. List all elements of H .
 - (b) List all cosets of H in G_{15} .
- (14) Let $R = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$. Show that R is a ring. Is R a field?
- (15) Let $f : B^3 \rightarrow B^5$ be a coding function given by $f(abc) = a\bar{a}b\bar{b}c$, where $\bar{a} = 1$ if $a = 0$ and $\bar{a} = 0$ if $a = 1$. What is the minimal distance between two nonzero codewords in B^5 ? How many errors can this code detect? How many errors can this code correct?
- (16) Write down the two-column decoding table for the code given by the generator matrix

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

Use this table to correct the message

010101 101010 001101 100101.