MAT126.R02: QUIZ 8

SOLUTIONS

Find the volume of the solid obtained by rotating the region bounded by the curves

$$y = x^3, \quad x = 0, \quad y = 1$$

about $x = 2$.

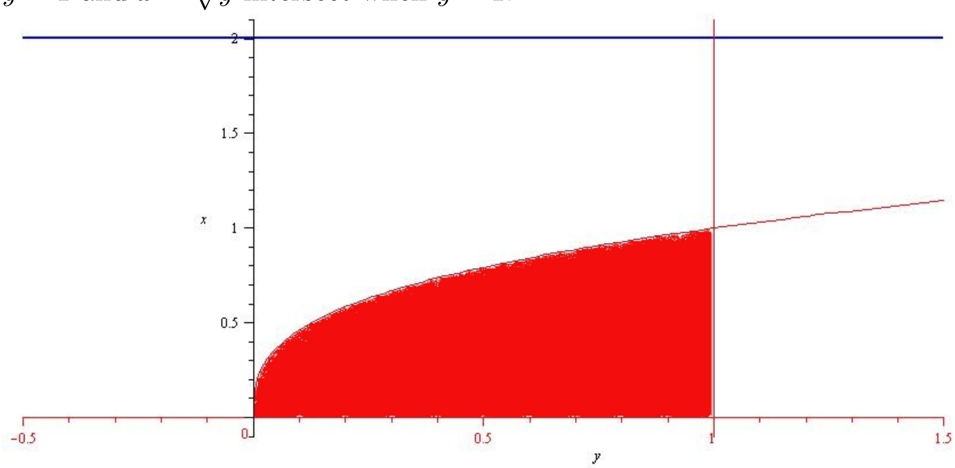
(Sketch the region first.)

The axis of revolution is vertical, so we need to rewrite the curves in terms of y:

$$x = \sqrt[3]{y}, \quad x = 0, \quad y = 1$$

Intersection points: $x = 0$ and $x = \sqrt[3]{y}$ intersect at the origin.

$y = 1$ and $x = \sqrt[3]{y}$ intersect when $y = 1$.



Since the region is "below" the axis of revolution, the inner shell is formed by $x = \sqrt[3]{y}$ and the outer shell by $x = 0$.

The volume is $\int_0^1 \pi \left((0-2)^2 - (\sqrt[3]{y}-2)^2\right) dy = \pi \int_0^1 4 - (y^{2/3} - 4y^{1/3} + 4) dy =$

$$\pi \left(-\frac{y^{5/3}}{5/3} + 4\frac{y^{4/3}}{4/3}\right)\Bigg|_0^1 = \pi\left(-\frac{1}{5/3} + 4\frac{1}{4/3}\right) = \frac{12\pi}{5}$$