

MAT126.R01: QUIZ 8

SOLUTIONS

Find the area of the region bounded by the curves

$$y = x^3 - 2x, \quad y = x^2.$$

(Draw the region first.)

Intersection points: $x^3 - 2x = x^2$

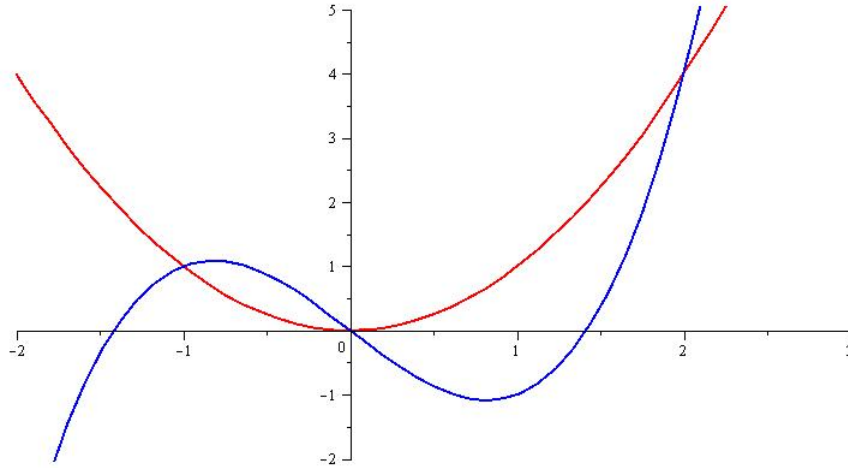
$$x^3 - x^2 - 2x = 0$$

$$x(x^2 - x - 2) = 0$$

Either $x = 0$ or $x^2 - x - 2 = 0$. The second equation has solutions $x = -1, 2$.

So, the intersection points are at $x = -1, 0, 2$

Between -1 and 0 , $y = x^3 - 2x$ is above $y = x^2$ (check their relative positions for, say, $x = -1/2$). Between 0 and 2 , $y = x^2$ is above.



$$\begin{aligned} \text{The area is } & \int_{-1}^0 (x^3 - 2x - x^2) dx + \int_0^2 (x^2 - (x^3 - 2x)) dx = \left(\frac{x^4}{4} - x^2 - \frac{x^3}{3} \right) \Big|_{-1}^0 + \\ & \left(\frac{x^3}{3} - \frac{x^4}{4} + x^2 \right) \Big|_0^2 = \left(0 - \frac{1}{4} + 1 - \frac{1}{3} \right) + \left(\frac{8}{3} - \frac{16}{4} + 4 - 0 \right) = \frac{47}{12} \end{aligned}$$