MAT126.R02: QUIZ 7

SOLUTIONS

Let $f(x) = \dfrac{2}{x^3}$ if $x \geq 1$ and $f(x) = 0$ if $x < 1$.

(a) Verify that $f(x)$ is a probability density function.

(1) *$f(x)$ is integrable because it is continuous everywhere except at $x = 1$ and has a jump discontinuity at $x = 1$;*

(2) *$f(x)$ is non-negative because $2/x^3$ is positive for all $x \geq 1$.*

(3) $\displaystyle\int_{-\infty}^{\infty} f(x)dx = \int_1^{\infty} \frac{2}{x^3}dx = \lim_{t\to\infty}\int_1^t \frac{2}{x^3}dx = \lim_{t\to\infty} 2\frac{x^{-2}}{-2}\bigg|_1^t = \lim_{t\to\infty} -t^{-2} - (-(1)^{-}2) = 1$

(b) Find $P(1 \leq X \leq 2)$.

$$P(1 \leq X \leq 2) = \int_1^2 f(x)dx = \int_1^2 \frac{2}{x^3}dx = 2\frac{x^{-2}}{-2}\bigg|_1^2 = -(2)^{-2} - (-(1)^{-2}) = -\frac{1}{4} + 1 = \frac{3}{4}$$