MAT126.R01: QUIZ 7

1. Evaluate the integral $\displaystyle\int_0^{2\pi} x \sin x \; dx$

$\displaystyle\int_0^{2\pi} x \sin x \; dx = x(-\cos x)\big|_0^{2\pi} - \int_0^{2\pi} -\cos x \; dx = -x\cos x\big|_0^{2\pi} + \sin x\big|_0^{2\pi} =$
$-(2\pi(\cos 2\pi) - 0\cos 0) + (\sin 2\pi - \sin 0) = -2\pi + 0 = -2\pi$
(integration by parts with $u = x$, $du = dx$, $dv = \sin x \; dx$, $v = -\cos x$)

2. Approximate the value of $\displaystyle\int_0^{2\pi} x\sin x \; dx$ using the trapezoidal rule with $n = 4$.

$\Delta x = \dfrac{2\pi - 0}{4} = \dfrac{\pi}{2}$

Endpoints: $0,\ 0 + \dfrac{\pi}{2} = \dfrac{\pi}{2},\ \dfrac{\pi}{2} + \dfrac{\pi}{2} = \pi,\ \pi + \dfrac{\pi}{2} = \dfrac{3\pi}{2},\ 2\pi.$

$T_4 = \dfrac{\pi/2}{2}\left(0\sin 0 + 2\dfrac{\pi}{2}\sin\dfrac{\pi}{2} + 2\pi\sin\pi + 2\dfrac{3\pi}{2}\sin\dfrac{3\pi}{2} + 2\pi\sin 2\pi\right) =$

$\dfrac{\pi}{4}\left(0 + 2\dfrac{\pi}{2} + 0 - 2\dfrac{3\pi}{2} + 0\right) = \dfrac{\pi}{4}(\pi - 3\pi) = -\dfrac{\pi(2\pi)}{4} = -\dfrac{\pi^2}{2}$