MAT126.R02: QUIZ 6

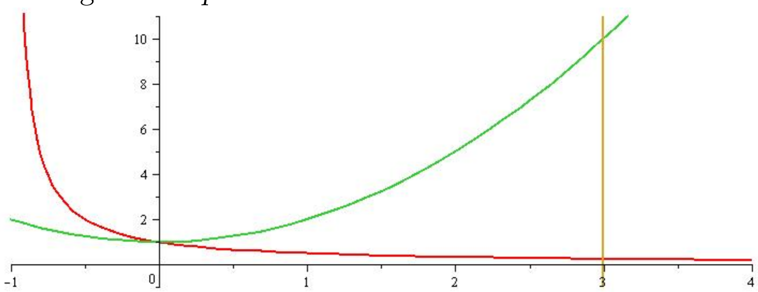
SOLUTIONS

Find the are of the region enclosed by the curves

$$y = \frac{1}{x+1}, \quad y = x^2 + 1, \quad x = 3.$$

(Draw the region first.)

The region is depicted below:



The right boundary of the region is given by $x = 3$.

The left boundary should be the intersection point between the two graphs.

This can be found either by realizing that $x^2 + 1 = \frac{1}{x+1}$ when $x = 0$ or by solving the algebraic equation

$x^2 + 1 = \frac{1}{x+1}$

Then $(x^2 + 1)(x + 1) = 1$, $x^3 + x^2 + x + 1 = 1$, $x^3 + x^2 + x = 0$, $x(x^2 + x + 1) = 0$, that is $x = 0$.

The graph of $y = x^2 + 1$ is above the graph of $y = \frac{1}{x+1}$ between 0 and 3. This can be either observed from the sketch of the region or by checking the values of both functions at any point between 0 and 3 (for instance, $x = 1$).

Thus the area is

$$\int_0^3 x^2 + 1 - \frac{1}{x+1}\,dx = \int_0^3 x^2 + 1\,dx - \int_0^3 \frac{1}{x+1}\,dx$$

The first integral computes as $\frac{x^3}{3} + x\Big|_0^3 = \frac{3^3}{3} + 3 = 12$

The second intergral is $\int_0^3 \frac{1}{x+1}\,dx = \int_1^4 \frac{1}{u} du = \ln u|_1^4 = \ln 4 - \ln 1 = \ln 4$

(here we use the substitution $u = x + 1, du = dx$)

Answer: $12 - \ln 4$.