

MAT126.R01: QUIZ 6

SOLUTIONS

Evaluate the following integrals:

$$(a) \int_1^e \frac{\ln^2 x}{x} dx = \int_0^1 u^2 du = \frac{u^3}{3} \Big|_0^1 = \frac{1}{3}$$

using the substitution $u = \ln x$, $du = \frac{1}{x} dx$; if $x = 1$, $u = \ln 1 = 0$, if $x = e$,
 $u = \ln e = 1$

$$(b) \int x \sin 2x dx = -\frac{1}{2} x \cos 2x - \int -\frac{1}{2} \cos 2x dx = -\frac{1}{2} x \cos 2x + \frac{1}{2} \int \cos 2x dx$$

integration by parts using $u = x$ and $dv = \sin 2x dx$, hence $du = dx$,
 $v = \int \sin 2x dx = -\frac{1}{2} \cos 2x + c$ (computed using the substitution $w = 2x$,
 $dw = 2dx$)

Then we compute $\int \cos 2x dx = \int \cos u \frac{1}{2} dx = \frac{1}{2} \sin u + C = \frac{1}{2} \sin 2x + C$
using the substitution $u = 2x$, $du = 2dx$

$$\text{Answer: } -\frac{1}{2} x \cos 2x + \frac{1}{2} \frac{1}{2} \sin 2x + C = \frac{1}{2} \left(\frac{\sin 2x}{2} - x \cos 2x \right) + C$$