MAT126.R02: QUIZ 5

SOLUTIONS

Consider the integral $\int_1^5 x \ln x \, dx$.

1. Use the trapezoidal rule with $n = 4$ to approximate this integral.

$\Delta x = \dfrac{4-1}{4} = 1$

Endpoints: 1, $1+1=2$, $2+1=3$, $3+1=4$, 5.

$T_4 = \dfrac{1}{2}(f(1) + f(2) + f(3) + f(4) + f(5)) =$

$\dfrac{1}{2}(1\ln 1 + 2\ln 2 + 3\ln 3 + 4\ln 4 + 5\ln 5) = \dfrac{2\ln 2 + 3\ln 3 + 4\ln 4 + 5\ln 5}{2}$

2. How large do we have to choose n to guarantee that the trapezoidal rule approximation of this integral is accurate to within 0.1?

The error E_T *is bounded by* $\dfrac{K(b-a)^3}{12n^2}$, *where* K *is a bound for the second derivative of* $f(x)$.

$f''(x) = (x\ln x)'' = \left(\ln x + x\frac{1}{x}\right)' = (\ln x + 1)' = \dfrac{1}{x}$

On the interval $[1,5]$ $\dfrac{1}{x}$ *is bounded by* $\dfrac{1}{1}$

$\dfrac{K(b-a)^3}{12n^2} = \dfrac{1(5-1)^3}{12n^2} = \dfrac{4^3}{12n^2} = \dfrac{16}{3n^2}$

If $\dfrac{16}{3n^2} < 0.1$, $n^2 > \dfrac{16}{3(0.1)} = \dfrac{160}{3}$

Hence $n > \sqrt{\dfrac{160}{3}}$.