MAT126.R02: QUIZ 4

SOLUTIONS

Evaluate the following integrals:

(a) $\int_1^4 \sqrt{x}\ \ln x\, dx = \ln x \frac{2x^{3/2}}{3}\Big|_1^4 - \int_1^4 \frac{2x^{3/2}}{3}\frac{1}{x}dx = \ln 4 \frac{2(4)^{3/2}}{3} - 0 - \frac{2}{3}\int_1^4 x^{1/2}dx =$
$\frac{16\ln 4}{3} - \frac{2}{3}\frac{2x^{3/2}}{3}\Big|_1^4 = \frac{16\ln 4}{3} - \frac{4}{9}\left(\frac{2(4)^{3/2}}{3} - \frac{2(1)^{3/2}}{3}\right) = \frac{16\ln 4}{3} - \frac{56}{27} =$
$\frac{144\ln 4 - 56}{27}$

integration by parts using $u = \ln x$ *and* $dv = \sqrt{x}dx$, *that is* $du = \frac{1}{x}dx$ *and* $v = \frac{2x^{3/2}}{3}$

(b) $\int \tan^{-1} x\, dx = x\tan^{-1} x - \int \frac{1}{1+x^2}x\, dx$

integration by parts using $u = \tan^{-1} x$ *and* $dv = dx$, *so that* $du = \frac{1}{1+x^2}dx$ *and* $v = x$

Then, $\int \frac{1}{1+x^2}x\, dx = \int \frac{1}{u}\frac{1}{2}du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+x^2| + C$

using the substitution $u = 1 + x^2$ *and* $du = 2xdx$

Answer: $x\tan^{-1} x - \frac{1}{2}\ln|1+x^2| + C$