MAT126.R01: QUIZ 1

Find expressions that estimate the area under the graph of

$$f(x) = x^3 + \cos x$$

from $x = 1$ to $x = 3$ using six approximating rectangles and
  (a) left endpoints;
  (b) right endpoints.
Do not evaluate the numerical expressions you obtain.

$$\Delta x = \frac{3-1}{6} = \frac{1}{3}$$

Dividing the interval $[1,3]$ into 6 parts, we obtain endpoints

$x_0 = 1,$

$x_1 = 1 + \dfrac{1}{3} = \dfrac{4}{3},$

$x_2 = 1 + 2\dfrac{1}{3} = \dfrac{5}{3},$

$x_3 = 1 + 3\dfrac{1}{3} = 2,$

$x_4 = 1 + 4\dfrac{1}{3} = \dfrac{7}{3},$

$x_5 = 1 + 5\dfrac{1}{3} = \dfrac{8}{3},$

$x_6 = 3.$

(a) $L_6 = \Delta x(f(x_0) + f(x_1) + \cdots + f(x_5)) =$

$$\frac{1}{3}\left(1^3 + \cos 1 + \left(\frac{4}{3}\right)^3 + \cos\frac{4}{3} + \left(\frac{5}{3}\right)^3 + \cos\frac{5}{3} + 2^3 + \cos 2 + \left(\frac{7}{3}\right)^3 + \cos\frac{7}{3} + \left(\frac{8}{3}\right)^3 + \cos\frac{8}{3}\right)$$

(b) $R_6 = \Delta x(f(x_1) + f(x_2) + \cdots + f(x_6)) =$

$$\frac{1}{3}\left(\left(\frac{4}{3}\right)^3 + \cos\frac{4}{3} + \left(\frac{5}{3}\right)^3 + \cos\frac{5}{3} + 2^3 + \cos 2 + \left(\frac{7}{3}\right)^3 + \cos\frac{7}{3} + \left(\frac{8}{3}\right)^3 + \cos\frac{8}{3} + 3^3 + \cos 3\right)$$