MAT126.R01: QUIZ 0

SOLUTIONS

If you could not even start on any of the derivatives in problem 4, you should seriously consider dropping this course.

1. $\ln\left(\sin\frac{\pi}{2}\right) = \ln(1) = 0$

2. Solve for x: $3^{x^2+x} = 9$
$3^{x^2+x} = 3^2$
$x^2 + x = 2$
$x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$
$x = -2, 1$

3. (a) $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2} x+2 = 2+2 = 4$

(b) $\lim_{x\to 0} \frac{x^2-4}{x-2} = \lim_{x\to 0} \frac{0^2-4}{0-2} = \frac{-4}{-2} = 2$

4. Differentiate the following functions:
(a) $(e^{\sin x})' = e^{\sin x}\cos x$
using the chain rule with $u = \sin x$, $(e^u)' = e^u$, $(\sin x)' = \cos x$

(b) $(t^5 \ln t)' = (t^5)' \ln t + t^5 (\ln t)' = 5t^4 \ln t + t^5 \frac{1}{t} = 5t^4 \ln t + t^4 = (5\ln t + 1)t^4$
(using product rule)

(c) $(\sqrt[3]{w+1} + \sqrt[3]{w-1})' = ((w+1)^{1/3} + (w-1)^{1/3})' = \frac{1}{3}(w+1)^{-2/3} + \frac{1}{3}(w-1)^{-2/3} = \frac{1}{3}\left(\frac{1}{\sqrt[3]{(w+1)^2}} + \frac{1}{\sqrt[3]{(w-1)^2}}\right)$

5. Compute $\int_{-\pi}^{\pi} x\cos x \, dx$

$x\cos x$ is an odd function ($f(-x) = -f(x)$), so $\int_{-\pi}^{0} x\cos x\, dx = -\int_{0}^{\pi} x\cos x\, dx$, as the values of the function over the two intervals of integration are the opposites of each other. Hence the intergral from $-\pi$ to π is zero.