MAT 126: PRACTICE FOR MIDTERM II

SOLUTIONS

Chapter 5 Review Exercises

15. $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_1^2 \frac{1}{u}\,du = \frac{1}{2}\ln u|_1^2 = \frac{1}{2}(\ln 2 - \ln 1) = \frac{\ln 2}{2}$
using the substitution $u = x^2+1$, $du = 2x\ dx$ (hence, $x\ dx = \frac{1}{2}du$) and $u = 1$ when $x = 0$, $u = 2$ when $x = 1$

19. $\displaystyle\int_0^1 e^{\pi t}\,dt = \frac{1}{\pi}\int_0^{\pi} e^u\,du = \frac{1}{\pi}e^u|_0^{\pi} = \frac{1}{\pi}(e^{\pi} - e^0) = \frac{e^{\pi}-1}{\pi}$
using the substitution $u = \pi t$, $du = \pi\ dt$ (hence, $dt = \frac{1}{\pi}du$) and $u = 0$ when $t = 0$, $u = \pi$ when $x = 1$

21. $\displaystyle\int \frac{x+2}{\sqrt{x^2+4x}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{2}\frac{u^{1/2}}{1/2} + C = u^{1/2} + C = \sqrt{x^2+4x} + C$
using the substitution $u = x^2+4x$, $du = (2x+4)dx$ (and $(x+2)dx = \frac{1}{2}du$)

22. $\displaystyle\int_1^2 x^3\ln x\,dx = \frac{x^4}{4}\ln x\Big|_1^2 - \int_1^2 \frac{x^4}{4}\frac{1}{x}\,dx = \left(\frac{2^4}{4}\ln 2 - \frac{1}{4}\ln 1\right) - \int_1^2 \frac{x^3}{4}\,dx =$
$\displaystyle 4\ln 2 - \frac{x^4}{16}\Big|_1^2 = 4\ln 2 - \frac{16}{16} + \frac{1}{16} = 4\ln 2 - \frac{15}{16}$
using integration by parts with $u = \ln x$, $v = x^4/4$, $du = (1/x)dx$, $dv = x^3dx$

24. $\displaystyle\int_0^5 ye^{-0.6y}\,dy = -\frac{5}{3}ye^{-0.6y}|_0^5 - \int_0^5 -\frac{5}{3}e^{-0.6y}dy = \frac{5}{3}\left(-ye^{-0.6y}|_0^5 + \int_0^5 e^{-0.6y}dy\right) =$
$\displaystyle\frac{5}{3}\left(-ye^{-0.6y} - \frac{5}{3}e^{-0.6y}\right)\Big|_0^5 = \frac{5}{3}\left(-\frac{20}{3}e^{-3} + \frac{5}{3}\right)$
using integration by parts with $u = y$, $v = -\frac{5}{3}e^{-0.6y}$, $du = dy$, $dv = e^{-0.6y}dy$. Here v is obtained from dv by $v = \int dv = \int e^{-0.6y}dy = -\frac{10}{6}\int e^w dw = -\frac{10}{6}e^w + C = -\frac{5}{3}e^{-0.6y} + C$ (uses substitution $w = -0.6y$).

27. $\displaystyle\int_1^4 x^{3/2}\ln x\,dx = \frac{2}{5}x^{5/2}\ln x|_1^4 - \int_1^4 \frac{2}{5}x^{5/2}\frac{1}{x}\,dx = \frac{2}{5}(4^{5/2}\ln 4 - 1^{5/2}\ln 1) - \frac{2}{5}\int_1^4 x^{3/2}\,dx =$
$\displaystyle\frac{64}{5}\ln 4 - \frac{2}{5}\frac{2}{5}x^{5/2}|_1^4 = \frac{64}{5}\ln 4 - \frac{124}{25}$
using integration by parts with $u = \ln x$, $v = \frac{2}{5}x^{5/2}$, $du = \frac{1}{x}dx$, $dv = x^{3/2}dx$

28. $\displaystyle\int \sin x \cos(\cos x)\ dx = -\int \cos u\ du = -\sin u + C = -\sin(\cos x) + C$
using the substitution $u = \cos x$, $du = -\sin x\ dx$

34. $\displaystyle\int_0^1 \frac{e^x}{1+e^{2x}} dx = \int_1^e \frac{1}{1+u^2} du = \tan^{-1} u|_1^e = \tan^{-1} e - \tan^{-1} 1 = \tan^{-1} e - \frac{\pi}{4}$
using the substitution $u = e^x$, $du = e^x\ dx$ (so that $e^{2x} = (e^x)^2 = u^2$), and $u = 1$ when $x = 0$, $u = e$ when $x = 1$

47. $\Delta x = 1/10$, $f(x) = \sqrt{1+x^4}$
(a) $T_10 = \frac{1}{20}[\sqrt{1} + 2\sqrt{1+0.1^4} + 2\sqrt{1+0.2^4} + 2\sqrt{1+0.3^4} + 2\sqrt{1+0.4^4} + 2\sqrt{1+0.5^4} + 2\sqrt{1+0.6^4} + 2\sqrt{1+0.7^4} + 2\sqrt{1+0.8^4} + 2\sqrt{1+0.9^4} + \sqrt{1+1^4}] \approx 1.090608$
(b) $M_10 = \frac{1}{10}[\sqrt{1+0.05^4} + \sqrt{1+0.15^4} + \sqrt{1+0.25^4} + \sqrt{1+0.35^4} + \sqrt{1+0.45^4} + \sqrt{1+0.55^4} + \sqrt{1+0.65^4} + \sqrt{1+0.75^4} + \sqrt{1+0.85^4} + \sqrt{1+0.95^4}] \approx 1.088840$
The function is concave up, so (a) is an overestimate (the trapezoids cover the area under the graph completely), while (b) is an underestimate (the excess in every rectanlge is less than the undercoverage)

49. $\displaystyle f''(x) = \left(\frac{4x^3}{2\sqrt{1+x^4}}\right)' = 2\left(\frac{x^3}{\sqrt{1+x^4}}\right)' = 2\frac{3x^2\sqrt{1+x^4} - x^3\frac{2x^3}{\sqrt{1+x^4}}}{1+x^4} = \frac{2(x^6+3x^2)}{(1+x^4)^{3/2}}$. The denominator is always greater than 1 and the maximum of the numerator on the interval $[0,1]$ is 8 (at $x = 1$). Hence on $[0,1]$, $|f''(x)| \le 8$.
(a) $\displaystyle |E_T| \le \frac{8(1-0)^3}{12 * 10^2} \approx 0.006667.$
In order for the error to be less than 0.00001, we need to choose n such that $\displaystyle\frac{8}{12n^2} < 0.00001$. Thus, $\displaystyle n^2 > \frac{8}{0.00012}$ and $n > 258.2$. It suffices to take $n = 259$.
(b) $\displaystyle |E_M| \le \frac{8(1-0)^3}{24 * 10^2} \approx 0.003334$
In order for the error to be less than 0.00001, we need to choose n such that $\displaystyle\frac{8}{24n^2} < 0.00001$. Thus, $\displaystyle n^2 > \frac{8}{0.00024}$ and $n > 182.6$. It suffices to take $n = 183$.

55. $\displaystyle\int_1^\infty \frac{1}{(2x+1)^3}\ dx = \lim_{t\to\infty} \int_1^t \frac{1}{(2x+1)^3}\ dx = \lim_{t\to\infty} \frac{1}{2}\int_3^{2t+1} \frac{1}{u^3}\ du = \lim_{t\to\infty} \frac{1}{2}\left.\frac{u^{-2}}{-2}\right|_3^{2t+1} = \lim_{t\to\infty}\left(-\frac{1}{4(2t+1)^3} + \frac{1}{36}\right) = \frac{1}{36}$
using the substitution $u = 2x+1$, $du = 2dx$, and $u = 3$ when $x = 1$, $u = 2t+1$ when $x = t$

57. $\displaystyle\int_{-\infty}^0 e^{-2x}\ dx$ is divergent because $e^{-2x} \ge 1$ on $[-\infty, 0]$

59. $\displaystyle\int_1^e \frac{dx}{x\sqrt{\ln x}} = \lim_{t\to 1^+}\int_t^e \frac{dx}{x\sqrt{\ln x}} = \lim_{t\to 1^+}\int_{\ln t}^1 \frac{du}{\sqrt{u}} = \lim_{t\to 1^+} \frac{u^{1/2}}{1/2}\bigg|_{\ln t}^1 = \lim_{t\to 1^+}(2-2\sqrt{\ln t}) = 2$

using the substitution $u=\ln x$, $du = \frac{1}{x}dx = \frac{dx}{x}$, and $u=\ln t$ when $x=t$, $u=1$ when $x=e$

61. $\displaystyle\frac{x^3}{x^5+2} < \frac{x^3}{x^5} = x^{-2}$. Since $\displaystyle\int_1^\infty x^{-2}\,dx$ converges, by Comparison Theorem $\displaystyle\int_1^\infty \frac{x^3}{x^5+2}\,dx$ converges as well.