AMS 102: PRACTICE FOR MIDTERM II

SOLUTIONS

16.17. We need to estimate the mean concentration of dieldrin in the population of male minke whales. We will use 95% confidence interval.

Assuming the distribution in the population is normal and the sample we have is a SRS, we have $\mu = 357 \pm 1.960\dfrac{50}{\sqrt{8}} \approx 357 \pm 34.65 = 322.35$ to 391.65 ng/g.

We are thus 95% confident that the mean dieldrin concentration among male minke whales is between 322.35 and 391.65 ng/g.

16.18. Let μ be the mean dieldrin concentration. We test $H_0 : \mu = 100$ ng/g versus $H_a : \mu > 100$ ng/g. (The alternative is one-sided because we are concerned only with concentrations that exceed the FDA limit.)

Assuming we have a normal distribution in the population and a SRS, with $\bar{x} = 357$ ng/g, the test statistic is $z = \dfrac{357 - 100}{50/\sqrt{8}} \approx 14.54$. This value is so large that clearly $P(z > 14.54)$ is extremely small, that is, significant.

This is overwhelming evidence against H_0; we conclude that the mean dieldrin level is above 100 ng/g.

16.19. For 80% confidence, the margin of error is $1.282\dfrac{50}{\sqrt{8}} \approx 22.6628$, while for 90% confidence, the margin is $1.645\dfrac{50}{\sqrt{8}} \approx 29.0798$. Therefore, the confidence intervals are 334.34 to 379.66 ng/g (at 80%) and 327.92 to 386.08 ng/g (at 90%). Margin of error grows with increasing confidence.

16.20. We need to estimate the mean age-20 IQ score for low birth weight males.

We will use 95% confidence interval. We also assume that the sample is a SRS and that the distribution in the population is normal. es We have $\mu = 87.6 \pm 1.96\dfrac{15}{\sqrt{113}} \approx 87.6 \pm 2.77 \approx 84.8$ to 90.4.

We are thus 95% confident that the mean age-20 IQ score for low birth weight males is between 84.8 and 90.4.

16.21. Let μ be the mean age-20 IQ score for this population. We test $H_0 : \mu = 100$ versus $H_a : \mu < 100$; the alternative is one-sided because we suspect that these babies have lower IQs.

We assume that the distribution is normal and that our sample is a SRS. The test statistic is $z = \dfrac{87.6 - 100}{15/\sqrt{113}} \approx -8.79$. This is far below zero, hence $P(z < -8.79)$ is approximately zero. The p-value is significant. There is overwhelming evidence to reject H_0.

We conclude that the male population with lower birth weight has age-20 IQ below 100.

16.24. (a) All given probabilities add up to 0.98. The remaining probability is thus 0.02.

(b) P(not Google)$= 1 - 0.66 = 0.34$.

16.26. (a) All probabilities are between 0 and 1 and sum up to 1.

(b) $X \leq 2$ means that a woman has 2 or less children. This is the sum of the first three probabilities, 0.711.

(c) $P(X < 2) = 0.367$ (the sum of the first two probabilities)

(d) The event is $X \geq 3$, that is $X > 2$ (more than two children). $P(X > 2) = 1 - P(X \leq 2) = 1 - 0.711 = 0.289$.

16.39. (a) All probabilities are greater than or equal to 0, and their sum is 1.

(b) Let R_1 be Taster 1s rating and R_2 be Taster 2s rating. Add the probabilities on the diagonal (upper left to lower right): $P(R_1 = R_2) = 0.03 + 0.08 + 0.25 + 0.20 + 0.06 = 0.62$.

(c) $P(R_1 > R_2) = 0.19$. This is the sum of the ten numbers in the "lower left" part of the table; the bottom four numbers from the first column, the bottom three from the second column, the bottom two from the third column, and the last number in the fourth column. These entries correspond to, e.g., "Taster 2 gives a rating of 1, and Taster 1 gives a rating more than 1."

$P(R_2 > R_1) = 0.19$; this is the sum of the ten numbers in the "upper right" part of the table. We could find it by summing up the number in the upper right part, as above. Or, we could note that $P(R_2 > R_1) = 1 - (P(R_1 = R_2) + P(R_1 > R_2))$.

16.45. (a) Here is the stemplot:

96	8
97	344
97	888889
98	0133
99	6
100	2

The point 100.2 is probably an outlier but otherwise the distribution is quite normal-looking.

(b) We wish to determine if the data backs up the traditional statement that the normal temperature is 98.6.

$H_0 : \mu = 98.6$; $H_a : \mu \neq 98.6$. Take $\alpha = 0.05$.

$\bar{x} = 98.203$, so the test statistic $z = \dfrac{98.203 - 98.6}{0.7/\sqrt{20}} = -2.54$. $P(z < -2.54) = 0.055$, greater than $\alpha/2 = 0.025$. The evidence is significant and we conclude that the mean temperature is different from 98.6.

16.53. Let A be the event "income = 1 million" and B be "income =100,000." Then "A and B" is the same as A, so $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)} = \dfrac{P(A)}{P(B)} = \dfrac{181,000/130,424,000}{11,415,000/130,424,000} = \dfrac{181,000}{11,415,000} \approx 0.01586$

16.61. Let T be the even "test is positive" and C, the event "Jason is a carrier." The test is never positive for non-carriers, so $P(C|T) = 1$.

21.27. We give a 95% confidende interval for μ, the mean cholesterol level for pets.

With 25 degrees of freedom, the interval is $\mu = 193 \pm 2.060) \dfrac{68}{\sqrt{26}} \approx 165.53$ to 220.47 mg/dl.

We are 95% confident that the mean cholesterol level for pets is between 165.53 and 220.47 mg/dl.

21.41. Let μ be the mean age of the first word (in months).

$H_0 : \mu = 12$, $H_a : \mu > 12$.

We may assume that the sample is a SRS despite a high outlier (at 26 months). In this case, $\bar{x} = 13$ and $s \approx 4.9311$ months. The test statistic is $t = \dfrac{13 - 12}{4.9311/\sqrt{20}} \approx 0.907$ with 19 degrees of freedom. Even at 10% significance level this is not enough to reject H_0.

If we drop the outlier, $\bar{x} \approx 12.3158$ and $s \approx 3.9729$ months. The test statistic is then $t \approx 0.346$ with 18 degrees of freedom. There is even less evidence to reject H_0.

Thus we cannot conclude that the mean age at first word is greater than one year.

21.43. At the 90% confidence interval we have:

Using all 20 children, we find $\bar{x} = 13$ and $s \approx 4.9311$ months. With 19 degrees of freedom, $t^* = 1.729$, so the confidence interval is $13 \pm 1.9064 \approx$ 11.09 to 14.91 months.

If we drop the outlier, then $\bar{x} \approx 12.3158$ and $s \approx 3.9729$ months, and $t^* = 1.734$ with 18 degrees of freedom. The interval is $12.3158 \pm 1.5804 \approx 10.74$ to 13.90 months.