RESEARCH STATEMENT

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My research lies in the areas of differential geometry, topology, and their interaction with mathematical physics. More specifically, I am interested in the geometry of string structures, or trivializations of degree 4 characteristic classes such as the Pontryagin class $\frac{p_1}{2}$. A great deal of information can be obtained by analyzing harmonic 3-forms which represent string structures on a principal bundle. While these differential forms are related to objects in theoretical physics, the choice of string structure is often ignored. It is hoped that by paying attention to the choice of string structure, we can relate topological invariants of string manifolds to geometry and analysis. In particular, it is hypothesized that under certain conditions, the elliptic cohomology theory tmf gives obstructions to positive Ricci curvature metrics. Also, in joint work with Hisham Sati, we hope to relate string and "fivebrane" structures to fields appearing in M-Theory.

1. String structures and harmonic forms on principal bundles

Just as a spin structure on a principal $SO(n)$-bundle $P \xrightarrow{\pi} M$ is a lift of the its classifying map from $f : M \to BSO(n)$ to $\widetilde{f} : M \to BSpin(n)$, a string structure on a $Spin(n)$-bundle is a lift of the classifying map to $BString(n)$. Here, $BString(n)$ is the homotopy fiber of the stable characteristic class $\frac{p_1}{2} : BSpin(n) \to K(\mathbb{Z}, 4)$. For $n \geq 5$, this corresponds to killing the third homotopy group of $Spin(n)$. The name *String* comes from its tentative relationship to string theory and loop space mathematics, which will soon be explained. While there are several constructions of $String(n)$ as a group [Sto, ST, BSCS, Hen], we will only assume its existence and use its homotopy type.

Proposition 1. *Let $P \xrightarrow{\pi} M$ be a principal $Spin(n)$-bundle.*

- *A string structure exists if and only if $\frac{p_1}{2}(P) = 0 \in H^4(M; \mathbb{Z})$.*
- *A choice of string structure, up to homotopy, is equivalent to a cohomology class $\mathcal{S} \in H^3(P; \mathbb{Z})$ that restricts to the standard class in $H^3(Spin(n); \mathbb{Z})$ on each fiber. We call $\mathcal{S}$ a string class.*
- *String classes are a torsor for $H^3(M; \mathbb{Z})$; any two are related by $\mathcal{S}' - \mathcal{S} \in \pi^* H^3(M; \mathbb{Z})$.*
- *We say a spin manifold M is string if $\frac{p_1}{2}(Spin(TM)) = 0$.*

Passing from $H^3(Spin(TM); \mathbb{Z}) \to H^3(Spin(TM); \mathbb{R})$, we lose torsion information but gain the ability to represent string classes by differential forms using Hodge theory. We now address the question of what the harmonic representative of a string class is.

In [Red1] and [Red2] we observe and prove that the harmonic forms on a principal bundle, in low dimensions, have a natural characterization in terms of harmonic forms on the base and Chern–Simons forms. To describe this result, we begin with the following data and notation:

Riemannian manifold	(M, g_M)	Harmonic k-forms on M	$\mathcal{H}^k(M)$
Compact Lie group	G	Principal G-bundle	$P \xrightarrow{\pi} M$
Connection on P	Θ	Curvature of Θ	Ω

The connection Θ picks out an equivariant horizontal distribution in TP and thus decomposes TP into horizontal and vertical vectors. Choosing some bi-invariant metric g_G on G then gives the local product metric on P

$$g_P := \pi^* g_M \oplus g_G.$$

The closed manifold P now has a Riemannian metric, and it is natural to investigate the harmonic forms with respect to this metric.

In concrete calculations, however, one observes that these harmonic forms have no obvious characterization, and they actually change under a global rescaling of the metric g_M. For this reason, we introduce a scaling factor $\delta > 0$ and define the metric on P

$$g_\delta := \delta^{-2}\pi^* g_M \oplus g_G.$$

The limit $\delta \to 0$, where the volume of the base is very large with respect to the volume of the fiber, is known as the adiabatic limit and appears in a number of constructions. In particular, the adiabatic limits of eta-invariants for Dirac operators appear when calculating holonomy of connections on determinant line bundles [Fre1]. The adiabatic limit of the Hodge Laplacian was dealt with in the work of Mazzeo–Melrose [MM], Dai [Dai], and Forman [For]. While the metric g_δ and Laplacian Δ_{g_δ} become singular at $\delta = 0$, their work proves that the space of harmonic k-forms $\operatorname{Ker} \Delta^k_{g_\delta}$ smoothly extends to $\delta = 0$. We use the notation

$$\mathcal{H}^k(P) := \lim_{\delta \to 0} \operatorname{Ker} \Delta^k_{g_\delta} \subset \Omega^k(P),$$

and note that $\mathcal{H}^k(P) \cong H^k(P;\mathbb{R})$ canonically. Furthermore, their work discovers a relationship between $\mathcal{H}^k(P)$ and the Leray–Serre spectral sequence for the fiber bundle $G \hookrightarrow P \to M$. Because of this relationship, showing that a form is in $\mathcal{H}^k(P)$ does not require constructing a δ-family of g_δ-harmonic forms and taking the limit. Instead, we only need to show that a given form is harmonic up to a low-order power of δ related to when the spectral sequence collapses. For a $Spin(n)$-bundle with $\frac{p_1}{2}(P) = 0 \in H^4(M;\mathbb{R})$, the spectral sequence calculating $H^3(P;\mathbb{R})$ collapses at $N = 2$ and makes our calculations possible.

The harmonic 1-forms on P are easy to describe for any compact connected G, and they are in fact independent of the parameter δ. For example,

Theorem 2. *Let $G = U(n)$, and $\frac{i}{2\pi}\operatorname{Tr}(\Theta)$ be the Chern–Simons 1-form. Then*

$$\mathcal{H}^1(P) = \begin{cases} \pi^*\mathcal{H}^1(M) & \text{if } c_1(P) \neq 0 \in H^2(M;\mathbb{R}) \\ \pi^*\mathcal{H}^1(M) \oplus \mathbb{R}[\frac{i}{2\pi}\operatorname{Tr}(\Theta) - \pi^* h] & \text{if } c_1(P) = 0 \in H^2(M;\mathbb{R}) \end{cases}$$

where $h \in \Omega^1(M)$ is the unique form such that $dh = \frac{i}{2\pi}Tr(\Omega) = c_1(P,\Theta)$ is the first Chern form and $h \in d^*\Omega^2(M)$.

When G is semisimple, $H^1(G;\mathbb{R}) = H^2(G;\mathbb{R}) = 0$, and the adiabatic-harmonic 1- and 2-forms are

$$\mathcal{H}^1(P) = \pi^*\mathcal{H}^1(M) \text{ and } \mathcal{H}^2(P) = \pi^*\mathcal{H}^2(M).$$

While the description of $\mathcal{H}^1(P)$ is actually independent of δ, the description of $\mathcal{H}^2(P)$ only holds when one considers the adiabatic limit.

To calculate $\mathcal{H}^3(P)$, we assume G is simple, and for notational simplicity we will use $G = Spin(n)$, but all of the statements hold for $SU(n)$ bundles once $\frac{p_1}{2}$ is replaced by c_2. Let

$$\alpha(\Theta) = \langle \Omega \wedge \Theta \rangle - \frac{1}{6}\langle \Theta \wedge [\Theta \wedge \Theta] \rangle \in \Omega^3(P)$$

be the Chern–Simons 3-form associated to the connection Θ (where $\langle \cdot, \cdot \rangle$ is a suitably normalized invariant inner product on $spin(n)$).

Theorem 3. *Let $G = Spin(n)$, (with $n = 3$ or $n \geq 5$). Then,*

$$\mathcal{H}^3(P) = \begin{cases} \pi^*\mathcal{H}^3(M) & \frac{p_1}{2}(P) \neq 0 \in H^4(M;\mathbb{R}) \\ \pi^*\mathcal{H}^3(M) \oplus \mathbb{R}[\alpha(\Theta) - \pi^* h] & \frac{p_1}{2}(P) = 0 \in H^4(M;\mathbb{R}) \end{cases}$$

where $h \in \Omega^3(M)$ is the unique form such that $dh = \langle \Omega \wedge \Omega \rangle = \frac{p_1}{2}(P, \Theta)$ and $h \in d^*\Omega^4(M)$.

In particular, the Chern–Simons 3-form $\alpha(\Theta)$ is in $\mathcal{H}^3(P)$ precisely when the Chern–Weil 4-form $\langle \Omega \wedge \Omega \rangle$ is identically 0. This naturally leads to the following questions I am currently working on.

Question 1.

(1) *Is there a characterization of $\mathcal{H}^k(P)$ for $k > 3$ in terms of Chern–Simons forms on P and harmonic forms on the base?*
(2) *Can one explicitly describe the ring structure of $\mathcal{H}^*(P)$? Given two forms in $\mathcal{H}^*(P)$, is the wedge product of the two, minus some canonical correction terms, an element of $\mathcal{H}^*(P)$?*

Remember that, from Proposition 1, string structures on a $Spin(n)$-bundle P correspond to string classes in $H^3(P;\mathbb{Z})$.

Proposition 4. *Given a metric g, connection Θ, and string class $\mathcal{S} \in H^3(P;\mathbb{Z})$, the canonical adiabatic-harmonic form $[\mathcal{S}] \in \mathcal{H}^3(P)$ representing $\mathcal{S}$ decomposes as*

$$[\mathcal{S}] = \alpha(\Theta) - \pi^* H_{g,\Theta,\mathcal{S}} \in \Omega^3(P),$$

where $H_{g,\Theta,\mathcal{S}} \in \Omega^3(M)$. Furthermore, the map

$$\{\text{String classes}\} \to \Omega^3(M)$$
$$\mathcal{S} \mapsto H_{g,\Theta,\mathcal{S}}$$

is equivariant under $H^3(M;\mathbb{Z})$, which acts on string classes by $\mathcal{S} \mapsto \mathcal{S} + \pi^* H^3(M)$, and acts on $\Omega^3(M)$ by the addition of a harmonic representative.

Therefore, once a metric and connection have been chosen, we can associate to any string structure $\mathcal{S}$ a canonical 3-form $H_{g,\Theta,\mathcal{S}}$. When $P = Spin(TM)$ and Θ is the Levi–Civita connection, we use the notation $H_{g,\mathcal{S}}$. In general, when the canonical 3-form $H_{g,\Theta,\mathcal{S}}$ is integrated on a 3-cycle and reduced mod $\mathbb{Z}$, it equals the Cheeger–Simons character evaluated on the cycle [Fre2]. In this way, the form $H_{g,\Theta,\mathcal{S}}$ acts as a trivialization of the Chern–Weil form and Cheeger–Simons character.

2. Relevance of string structures

String structures are the next higher structure after spin structures, and it is useful to remember the key role played by spin structures in the construction of the spinor bundle and Dirac operator. Additionally, the generalized cohomology theory KO has a spin orientation α that refines the $\widehat{A}$-genus; the index of the Dirac operator on an n-manifold *with a chosen spin structure* is an element in $KO^{-n}(pt)$. If the spin manifold M admits a metric of positive scalar curvature, then $\widehat{A}(M) = 0$ and $\alpha(M, s) = 0$ for any spin structure s [Hit].

The Witten genus $\phi_W(M)$, or Witten index, of an n-manifold is a topological invariant defined by Witten to be the S^1-invariant index of the (not mathematically rigorous) Dirac operator $\not{D}_{LM}$ on the free loop space LM [Wit]. While $\phi_W(M)$ is defined for any oriented manifold, the condition that M is string implies that $\phi_W(M)$ is an integral modular form of degree n, or weight $n/2$. In [Sto] Stolz conjectures that if M is a string manifold and admits a metric of positive Ricci curvature, then the Witten genus $\phi_W(M) = 0$. His heuristic argument is based on the assumption that there should be a Bochner–Weizenböck-type formula for the Dirac operator on LM such that positive

Ricci curvature on M implies $\mathrm{Ker}(\not{D}_{LM}) = 0$. There is also a generalized cohomology theory named topological modular forms, or tmf, and it can be thought of as the universal elliptic cohomology theory. This cohomology theory has a string orientation σ that refines the Witten genus ϕ_W [AHS], hinting that tmf should provide refined invariants related to index theory on loop spaces. The $\widehat{A}$ and Witten genera, along with their refinements α and σ, are bordism invariants and can be thought of in terms of the following two commutative diagrams. Here Ω_n^G is G-bordism, and MF_n denotes integral modular forms of degree n.

$$
\begin{array}{ccc}
 & & KO^{-n}(pt) \\
 & \overset{\alpha}{\nearrow} & \downarrow \\
\Omega_n^{Spin} & \xrightarrow{\widehat{A}} & \mathbb{Z}
\end{array}
\qquad\qquad
\begin{array}{ccc}
 & & tmf^{-n}(pt) \\
 & \overset{\sigma}{\nearrow} & \downarrow \\
\Omega_n^{String} & \xrightarrow{\phi_W} & MF_n
\end{array}
$$

A question which has motivated much of my work is: are the σ-invariants in tmf also an obstruction to positive Ricci curvature? The naive guess that $\sigma(M, \mathcal{S}) = 0$ for all string structures $\mathcal{S}$ cannot be true, but if we include a condition involving both the metric and choice of string structure, there is a working hypothesis that we will describe in a moment.

First, these diagrams have an interesting interpretation using quantum field theory. Stolz and Teichner have shown that $KO^{-n} \simeq \mathcal{EFT}_n$, where $\mathcal{EFT}_n$ is the space of supersymmetric 1-dimensional Euclidean Field Theories of degree n [ST]. Roughly speaking, one can interpret the map α as, up to homotopy, quantizing fermionic particles moving in a target Riemannian spin manifold M. The integer $\widehat{A}(M)$ is the partition function of the resulting quantum field theory [Wit]. If M^n admits a positive scalar curvature metric, the 1-dimensional field theory associated to M^n is qualitatively the same as the trivial field theory of degree n.

While the cohomology theory tmf is only defined via complicated topological methods, there are several attempts to give a geometric definition [Seg, ST, BDR, HK]. In particular, Stolz–Teichner conjecture that tmf^{-n}, the n-th space in the tmf spectrum, is homotopy equivalent to the space of 2-dimensional supersymmetric quantum field theories of degree n [ST]. In this view, the orientation σ corresponds to the quantization of a certain supersymmetric sigma model. The fields in the classical theory are maps $X : \Sigma \to M$ from a spin Riemann surface Σ to a target Riemannian spin manifold (M, g), together with a section of $K^{1/2} \otimes X^*TM$ where $K^{1/2}$ is a spinor bundle on Σ. The Witten genus $\phi_W(M)$ is the partition function of this quantum field theory.

The necessity of $\frac{p_1}{2}(M) = 0$ is easily seen in this context. In general, quantizing this sigma model is not mathematically well-defined due to the non-existence of a measure on $\mathrm{Map}(\Sigma, M)$. However, in the process of trying to quantize, one must trivialize a Pfaffian line bundle $\mathrm{Pf}(\not{D}_X) \to \mathrm{Map}(\Sigma, M)$ with connection [Wit, AS]. This line bundle is topologically trivial if $\frac{p_1}{2}(M) = 0$. In fact, the canonical 3-form $H_{g,\mathcal{S}}$ transgresses to a 1-form on $\mathrm{Map}(\Sigma, M)$ that trivializes the connection on $\mathrm{Pf}(\not{D}_X)$ [AS, Red3]. For this reason, the form $H_{g,\mathcal{S}}$ must be added to the classical action as a topological term.

Related to this, the vanishing of $\frac{p_1}{2}(M) = 0$ allows one to construct the spinor bundle on LM, despite analytic difficulties in defining the Dirac operator. When constructing the spinor bundle on LM, one needs to construct a line bundle with connection over LP. It is known that the curvature of such a connection must be the transgression of $\alpha(\Theta) - \pi^* H_{g,\mathcal{S}}$ to a 2-form on LP, plus another canonical term [CP].

3. Speculation on positive Ricci curvature and tmf

We now return to the conjecture of Stolz: if M is a string manifold and admits a positive Ricci curvature metric, then $\phi_W(M) = 0$. There are no known counterexamples to this conjecture, and it holds for certain classes of manifolds such as complete intersections and homogeneous spaces. However, the naive guess that the σ-invariant in $tmf^{-*}(pt)$ should also vanish is not true. There are a number of manifolds admitting positive Ricci curvature metrics that, with certain string structures, are mapped by σ to torsion classes in $tmf^{-*}(pt)$. For example, the standard metric on the 3-sphere S^3 has positive Ricci (and even sectional) curvature, yet σ maps the different string structures surjectively onto $tmf^{-3}(pt) \cong \pi_3^s \cong \mathbb{Z}/24$.

If the σ-invariant in tmf is related to $\not{D}_{LM}$ or the supersymmetric chiral sigma model, and we know that these physical constructions involve the string structure and form $H_{g,\mathcal{S}}$, then any attempt to generalize the Stolz conjecture to tmf must simultaneously consider the geometry of the manifold and the string structure.

Hypothesis. *If the n-manifold M with string structure $\mathcal{S}$ admits a metric g such that both $Ric(g) > 0$ and $H_{g,\mathcal{S}} = 0$, then $\sigma(M, \mathcal{S}) = 0 \in tmf^{-n}(pt)$.*

Heuristically, the hypothesis states that if one does not need to add the $H_{g,\mathcal{S}}$-term to the sigma model and (M, g) has positive Ricci curvature, then the resulting quantum field theory should be qualitatively the same as the trivial field theory.

This hypothesis has an equivalent formulation, as shown in [Red3]. Consider the metric connection $\nabla^{g,\mathcal{S}}$ with torsion tensor T determined by $g(T(X, Y), Z) = H_{g,\mathcal{S}}(X, Y, Z)$. Requiring both $Ric(g) > 0$ and $H_{g,\mathcal{S}} = 0$ is equivalent to requiring that the Ricci curvature of $\nabla^{g,\mathcal{S}}$ is positive when g is scaled to have small volume.

The conditions of the hypothesis are rather strong, and in particular it requires that the Chern–Weil form $\frac{p_1}{2}(M, g) = 0 \in \Omega^4(M)$. To verify the hypothesis explicitly for a manifold with string structure sent to a non-zero class in tmf, one must consider the entire space of metrics on M, which is quite difficult. However, in certain nicer classes of metrics, the hypothesis is true. For example, the hypothesis holds for all metrics and string structures in the 1-parameter family of Berger 3-spheres, where the metric is deformed by rescaling the fibers of the Hopf fibration. The Berger 3-spheres also show that the hypothesis would be false if $Ric(g) > 0$ were replaced by $Ric(g) \geq 0$ [Red3].

In dealing with the above hypothesis and attempting to relate it to the Stolz conjecture, one is naturally led to the following intermediate and interesting questions.

Question 2.

(1) *Given a $Spin(n)$-bundle P, let $\omega \in \Omega^4(M)$ be a closed form such that $[\omega] = \frac{p_1}{2}(M) \in H^4(M; \mathbb{R})$. Does there always exist a connection Θ on P such that $\frac{p_1}{2}(P, \Theta) = \omega$?*
(2) *For such an ω, if $P = Spin(TM)$ does there exist a Riemannian metric g such that the Levi–Civita connection satisfies $\frac{p_1}{2}(M, g) = \omega$?*
(3) *Given a spin manifold M with $\frac{p_1}{2}(M) = 0 \in H^4(M; \mathbb{Z})$, when can you find a Riemannian metric g such that the canonical form $H_{g,\mathcal{S}} = 0 \in \Omega^3(M)$ for some string structure $\mathcal{S}$?*

The first question has recently been answered affirmatively if one is allowed to add a trivial bundle [SS], but the unstable case was not considered. While it is not necessarily expected that these questions have affirmative answers, any obstructions would be mathematically interesting.

Also, while there is no general geometric/analytic interpretation for the σ-invariants, in dimension 3 this is precisely the Adams e-invariant for framed bordism, which can be calculated geometrically by bounding M^3 with a 4-dimensional spin Riemannian manifold W^4. The e-invariant then involves integrating the Chern–Weil $\frac{p_1}{2}$-form on W^4 and $H_{g,\mathcal{S}}$ on M^3.

Question 3. *Using knowledge of the e-invariant and the 3-dimensional spin-bordism category, can we use analytic estimates to prove the hypothesis on the entire space of metrics for certain 3-manifolds?*

4. String/Fivebrane Duality and M-Theory

We now describe a joint project with mathematical physicist Hisham Sati. The goal is to better understand the geometry of fivebrane structures and to relate string/fivebrane structures to fields appearing in M-theory. Just as we say a spin manifold M is string if $\frac{p_1}{2}(M) = 0$, a string manifold is fivebrane if $\frac{p_2}{6}(M) = 0 \in H^8(M;\mathbb{Z})$ [SSS].

One of the fields in M-theory is the C-field. At the level of supergravity, the C-field is a 3-form C_3. However, upon quantization, it has been observed that there is more structure, and the following E_8 model was obtained in [DMF]. Assume Y^{11} is an 11-dimensional spin manifold, and $P \to Y^{11}$ is a principal E_8-bundle. The C-field can be viewed as the pair $(\Theta_{E_8}, c) \in \mathcal{A}(P) \times \Omega^3(Y^{11})$, and it is given in terms of the Chern–Simons 3-forms for P and $Spin(TY^{11})$ and the closed form $c \in \Omega^3(Y^{11})$:

$$C = \alpha(\Theta_{E_8}) - \frac{1}{2}\alpha(\Theta_{Spin}) + c.$$

The form C satisfies $dC = G_4(\Omega_{E_8}, \Omega_{Spin})$, where $G_4 = a - \frac{1}{4}p_1$, and a is the generator of $H^4(BE_8, \mathbb{Z})$. The Bianchi identity and equation of motion for the C-field in M-theory are

$$dG_4 = 0$$
$$\frac{1}{\ell_p^3} d * G_4 = \frac{1}{2} G_4 \wedge G_4 - I_8,$$

where ℓ_p is a constant, I_8 is the polynomial $I_8 = \frac{p_2 - (\frac{1}{2}p_1)^2}{48}$, and $*G_4$ as the dual C-field. In particular, the degree 8 class $\frac{1}{2}G_4 \wedge G_4 - I_8$ must vanish at the level of integral cohomology.

Question 4.

(1) *What is the relationship between string structures and the C-field?*
(2) *What is the relationship between fivebrane structures and the dual C-field?*
(3) *Can the duality duality between the C-field and its dual be expressed in terms of a duality between string and fivebrane structures?*

To answer these questions, we hope to view the C-field in terms of harmonic 3-forms on a bundle, as described earlier, and interpret the dual C-field in terms of harmonic 7-forms, as discussed in Question 1. We then hope to express the duality in terms of a duality between the string and fivebrane structures.

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