Math 303 Fall 2007 Midterm 2 Review

On the midterm, you may have one $8\frac{1}{2}$ x 11 sheet of paper (one-side) with formulas, notes, and examples (or favorite recipes or ...). There are no calculators allowed, and you may not use any other material, including books, homework assignments, or class notes. All questions will be partial credit. It will cover 3.1-3.7 and 4.1 with the emphasis on 3.1-3.6. Notice that there is some overlap in the following problems.

1. (3.1: 20-26, 3.2: 1-12) Show whether the following sets of sets of functions are linearly dependent or independent:
a. $\{\cos 2x, \sin 2x\}$
b. $\{\cos^2 x, \sin^2 x\}$
c. $\{\cos^2 x, \sin^2 x, 4\}$.

2. (3.1: 1-16, 3.2: 13-20) Verify that the following functions are solutions to the given differential equation. Solve the initial value problem. (Bonus: What does the Wronskian of your solutions have to do with solving the initial value problem?)
a. $x^2y'' + 2xy' - 6y = 0; y_1 = x^2, y_2 = x^{-3}; y(1) = 3, y'(1) = 1$
b. $y^{(3)} - 6y'' + 11y' - 6y = 0; y_1 = e^x, y_2 = e^{2x}, y_3 = e^{3x};$
$y(0) = 0, y'(0) = 1, y''(0) = 3.$

3. (3.1: 33-48, 3.3: 1-20, 33-36) Find general solutions to the following homogeneous equations.
a. $3y' - y = 0$
b. $y'' + 2y' - 15y = 0$
c. $9y^{(3)} + 12y'' + 4y' = 0$
d. $y^{(4)} - 8y'' + 16y = 0$

4. (3.1: 1-16, 3.3: 21-26) Solve the following initial value problems.
a. $3y' - y = 0; y(0) = 5$
b. $y'' + 2y' - 15y = 0; y(0) = 0, y'(0) = 4$
c. $9y^{(3)} + 12y'' + 4y' = 0; y(0) = 0, y'(0) = 1, y''(0) = \frac{10}{3}$

5. Show that the function $y = \sin x$ satisfies the equation

$$yy'' - (y')^2 = -1.$$

Does it then follow that $y = 3\sin x$ is also a solution? Why or why not?

6. (3.2: 21-24, 3.5: 1-20, 31-40) Find a general solution to the following nonhomogeneous equations and/or solve the initial value problem.
a. $y'' + y = 3x; y(0) = 2, y'(0) = -2$
b. $y'' + 9y = 2\cos 3x + 3\sin 3x$

7. (3.5:21-30) Set up the appropriate form of a particular solution y_p, but do not determine the values of the coefficients.
a. $y'' - 2y' + 2y = e^x \sin x$

8. (3.4: 24-33) Show that if a mass-spring-dashpot system with no external force is underdamped (i.e. $c^2 < 4km$), then the mass passes through the equilibrium position an infinite number of times.

9. (3.4: 15-21) Find the position function for the following mass-spring-dashpot system. What happens to the position for large time?
a. $m = \frac{1}{2}, c = 3, k = 4; x_0 = 2, v_0 = 0$

10. (3.4: 1-4) What is the amplitude and period for the undamped mass-spring system with $m = 2, k = 8, x_0 = 3, v_0 = 8$?

11. (3.6: 1-14)a. Find the position function for the undamped mass-spring system with external force $4e^{-t}$ given by the equation

$$x'' + x = 4e^{-t}; x_0 = 3, v_0 = -2.$$

b. Find the amplitude of the steady-periodic solution to

$$mx'' + kx = F_0 \cos(\omega t).$$

12. (3.6: 15-18) In the following mass-spring-dashpot systems with external force $F_0 \cos(\omega t)$, is there practical resonance for some $\omega > 0$? If so, at what frequency ω will this occur? (Hint: The formula for the amplitude of the steady-periodic solution is

$$C(\omega) = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}.$$

a. $2x'' + \sqrt{2}x' + 5x = F_0 \cos(\omega t)$.

13. (4.1: 1-10) Transform the mass-spring-dashpot equation $mx'' + cx' + kx = 0$ into a system of first-order equations.