

## Math 303 Fall 2007 Final Exam Review

The final exam will be cumulative with a slight emphasis on the material covered since the second midterm. The best way to review is to work through the first two midterms and homework-style problems.

On the final, you may have one  $8\frac{1}{2}$  x 11 sheet of paper (one-side) with formulas, notes, and examples. The cheat-sheet should be made by you and not merely copied from another student. There are no calculators allowed, and you may not use any other material, including books, homework assignments, or class notes. All questions will be partial credit.

The review sheets for Midterms 1-2 along with the following problems provide a good idea of what kind of questions to expect.

1. a. Show directly (by plugging in solution  $x(t)$ ) that  $x(t) = e^{\lambda t}$  is a solution to  $ax'' + bx' + cx = 0$  if and only if  $\lambda$  is a solution to  $a\lambda^2 + b\lambda + c = 0$ .

b. Show directly that  $\vec{x}(t) = \vec{v}e^{\lambda t}$  is a solution to  $\vec{x}'(t) = A\vec{x}(t)$  if and only if  $\vec{v}$  is an eigenvector of  $A$  with eigenvalue  $\lambda$ .

c. Suppose that  $\vec{v}_1$  is an eigenvector of  $A$  with eigenvalue  $\lambda$ . If  $\vec{x}(t) = (\vec{v}_1 t + \vec{v}_2)e^{\lambda t}$  is a solution to  $\vec{x}'(t) = A\vec{x}(t)$ , then determine an explicit relationship between  $\vec{v}_1$  and  $\vec{v}_2$  (derive the equation for a chain of generalized eigenvectors).

2. Solve the second-order linear equation

$$x'' + 5x' + 6x = 0$$

(a) by using the methods from Chapter 3 (use characteristic equation)  
(b) by transforming it into a system of 2 first-order equations and using the techniques of Chapter 5.

3. Find 2 linearly independent solutions to

$$\vec{x}' = \begin{bmatrix} 3 & -1 \\ 5 & 3 \end{bmatrix} \vec{x}.$$

Show that they are linearly independent. Write a general solution.

4. Solve the initial value problem

$$\begin{cases} \frac{dx}{dt} = 4x + 2y \\ \frac{dy}{dt} = 3x - y \end{cases} ; x(0) = 3, y(0) = -2.$$

5. Find general solutions and write a fundamental matrix solution to  $\vec{x}' = A\vec{x}$  for the following matrices

$$A = \begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 1 & -1 \\ -4 & -3 & -1 \\ 4 & 4 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 2 \\ -5 & -3 & -7 \\ 1 & 0 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 2 & 0 & 0 \\ -7 & 9 & 7 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 1 \\ -2 & -4 & -1 \end{bmatrix}.$$

6. Calculate  $e^{At}$  and use this to solve the initial value problem  $\vec{x}' = A\vec{x}$  with  $\vec{x}(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ , where

$$A = \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}.$$

7. Suppose that we have a two brine tank system with constant flow rate of 10 gal/min going into and out of both tanks. If the volume of tank I is 50gal and that of tank II is 25 gal, find the amount of salt in both tanks as a function of time, assuming that the original amount in tank I is 15 gal and that of tank II is 0 gal.

8. Find equilibrium solutions and determine their stability in the systems:

$$\begin{cases} \frac{dx}{dt} = -2x + y \\ \frac{dy}{dt} = x - 2y \end{cases}, \begin{cases} \frac{dx}{dt} = 1 - y^2 \\ \frac{dy}{dt} = x + 2y \end{cases}$$