

Name:

ID:

# Math 205: Midterm 1

## March 6, 2008

The exam is all partial credit. Please write neatly and clearly, showing all of your work. No calculators, cell phones, books, or notes may be used. The test contains 100 possible points. Good luck!

1	
2	
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Total	

1. (20 points) Let  $\mathbf{a} = (0, 3, -1)$ ,  $\mathbf{b} = (2, -1, 1)$ ,  $\mathbf{c} = (1, 1, 2)$ .

a. Calculate  $(3\mathbf{a} + \mathbf{b}) - \mathbf{c}$ .

b. What is  $\mathbf{a} \cdot \mathbf{b}$ ?

c. Find the angle between the two vectors  $\mathbf{b}$  and  $\mathbf{c}$ .

d. Find a vector perpendicular to both  $\mathbf{a}$  and  $\mathbf{c}$ .

2. (15 points) Find the distance between the point  $(5, -1, 2)$  and the plane  $2x - 2y + z = 7$ .

3. (15 points) Let  $\mathbf{r}(t) = t^2\mathbf{i} - (t + 2)\mathbf{j} + e^t\mathbf{k}$  be a curve in  $\mathbb{R}^3$ . Write an equation for the tangent line to the curve at  $t = 3$ .

4. (10 points) Consider the surface given by the equation

$$z = y - 2x^2.$$

- Find the level curve at an arbitrary height and then sketch several sample level curves (e.g. curves for  $c = -1, 0, 1, 2$  is sufficient).
- Then, use this information to give a *rough* sketch of the surface.



5. (10 points) Evaluate the following limits. If they fail to exist, explain why.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + 1} = ?, \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2} = ?$$

6. (15 points) Consider the surface given by the function

$$z = f(x, y) = (x + y)e^{2y}.$$

- a. Find  $\nabla f$  at the point  $(2, 2)$ .
- b. Write an equation for the tangent plane at the point  $(2, 2)$ .
- c. Using linear approximation, estimate  $f(1.8, 2.1)$ .

7. (15 points) Let  $\mathbf{P}(r, \theta) = (r \cos \theta, r \sin \theta)$  be a function  $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ . Notice that  $\mathbf{P}$  converts polar coordinates to rectangular coordinates.
- Calculate  $D\mathbf{P}$ .
  - Suppose  $f(x, y)$  is a function  $\mathbb{R}^2 \rightarrow \mathbb{R}$ , and at the point  $\mathbf{x}_0 = (0, 2)$ ,

$$\frac{\partial f}{\partial x}(\mathbf{x}_0) = -3, \quad \frac{\partial f}{\partial y}(\mathbf{x}_0) = 4.$$

Then, what is  $\frac{\partial f}{\partial r}$  and  $\frac{\partial f}{\partial \theta}$  at this same point? (Hint: consider  $D(f \circ \mathbf{P})$ , and convert the point  $\mathbf{x}_0$  to polar coordinates).