Math 205 Midterm 2 Review

The exam will take place in class on Thursday, April 10, 2008. No books, notes, or calculators are allowed. You will be held explicitly responsible for the material we have covered since the last midterm, though much of that depends implicitly on earlier material. Roughly speaking, the midterm covers information from sections 4.1-4.4, 5.1-5.3. The following is a rough summary of the concepts covered, followed by sample problems.

- Differentials and Taylor's Theorem: Creating linear and quadratic approximations to multivariable functions.
- Local extrema of functions (finding critical points, classifying them local mins/maxes, and saddles).
- Global extrema of functions on bounded regions.
- Extrema of functions with (1 or more) constraints. In particular, using Lagrange multipliers.
- Extrema in physical situations. You should be able to translate a real-life problem into a mathematical question (e.g. find equilibrium points in a physical situation with a conservative vector field.)
- Double integrals over general regions $D \subset \mathbb{R}^2$.
- Interpret double integrals in terms of volume.
- Change order of integration on double integrals.

1. a. Find the first-order Taylor polynomial for $f(x,y) = e^{2x}\cos y$ at the point $(0,0)$.

b. Find the second-order Taylor polynomial for $f(x,y) = e^{2x}\cos y$ at the point $(0,0)$.

c. Use a second-order Taylor approximation to estimate $e^{.2}\cos .2$.

2. Let $f(x,y) = x^2 + xy + y^2 + 2x - 2y + 5$. Find all critical points of f and determine their nature (local min, max, saddle).

3. Suppose the temperature of a space is given by the function

$$T(x,y,z) = e^{-y}(x^2 - y^2).$$

Find the hottest point on the ball $x^2 + y^2 + z^2 \leq 4$.

4. Suppose a particle is constrained to live on the surface

$$z = -xye^{-x^2-y^2}$$

and moves under the influence of a gravitational force (given by the potential energy function $V(x,y,z) = mgz$. Find any equilibrium points.

5. Find the minimum distance from the origin to the surface $x^2-(y-z)^2 = 1$.

6. Suppose a coffee shop sells beans from Arabia and Hawaii. If the Arabian beans are priced at x dollars per pound and the Hawaiian beans are priced at y dollars per pound, then research estimates that each week approximately $80 - 100x + 40y$ pounds of Arabian beans will be sold and $20 + 60x - 35y$ pounds of Hawaiian beans will be sold. The beans cost the coffee house \$2/lb for the Arabian and \$4/lb for the Hawaiian beans. How should the owners price the coffee beans in order to maximize their profits?

7. Find the volume of the region bounded by the planes

$$x = 0, x = \pi, y = 1, y = 2, z = 0, z = y\sin x.$$

8. Evaluate $\int\int_D 3y dA$, where D is the region bounded by $y = x^2 + 2$ and $y = 2x^2 - 2$.

9. Evaluate the following integral by switching the order of integration:

$$\int_0^\pi \int_y^\pi \frac{\sin x}{x}\, dx\, dy.$$