Math 205 Midterm 1 Review

The exam will take place in class on Thursday, March 6, 2008. No books, notes, or calculators are allowed. You will be responsible for the material we have covered so far. While we have roughly covered 1.1-3.2, there are a number of subtopics the book discusses that we have ignored. If you have not seen any homework problems on these, you will not be accountable for them (e.g. Newton's method, delta-epsilon proofs, inverse function theorem, curvature formulas are all things we skipped over). The following is a list of useful formulas/equations and sample problems.

Vector equations and properties:

$$\mathbf{a}\cdot\mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\|\cos\theta = a_1b_1 + \cdots + a_nb_n$$

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}\cdot\mathbf{a}}$$

$$\mathbf{a}\times\mathbf{b} = det\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

$$\|\mathbf{a}\times\mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\|\sin\theta$$

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\right)\mathbf{a}$$

Parametric equation for line passing through the point $\mathbf{x_0}$ with direction $\mathbf{b}$

$$\mathbf{x}(t) = \mathbf{x_0} + t\mathbf{b}$$

Equation for plane with normal vector $\mathbf{n}$ through the point $\mathbf{x_0}$

$$\mathbf{n}\cdot(\mathbf{x} - \mathbf{x_0}) = 0$$

Parametric equation for the plane spanned by the two vectors $\mathbf{a}, \mathbf{b}$ and containing the point $\mathbf{x_0}$

$$\mathbf{x}(s,t) = s\mathbf{a} + t\mathbf{b} + \mathbf{x_0}$$

Differentiation formulas:

$$f : \mathbb{R}^n \to \mathbb{R}, \qquad \nabla f = \left(\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}\right)$$

$$f : \mathbb{R}^n \to \mathbb{R}, \qquad Df = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \cdots & \frac{\partial f}{\partial x_n} \end{bmatrix}$$

$$\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m, \qquad D\mathbf{f} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

Chain Rule: $\quad D(\mathbf{f}\circ\mathbf{u}) = D\mathbf{f}(\mathbf{u})D\mathbf{u}$

Specific case: $$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

Sample problems:

1. Let $\mathbf{a} = (1, 0, -4), \mathbf{b} = (3, 2, 4), \mathbf{c} = (-2, -3, 1)$. Compute $\mathbf{a} - 2\mathbf{b}$, the length of $\mathbf{a}$, $(\mathbf{a} - 2\mathbf{b}) \cdot \mathbf{c}$, and $\mathbf{b} \times \mathbf{c}$.

2. What is the distance between the two points $(1, 2, 4)$ and $(-1, 1, 6)$? Write an equation for a line passing through those two points.

3. Find the angle between the two vectors $\mathbf{a} = (1, 1, 0)$ and $\mathbf{b} = (0, 1, -1)$. Find a vector of length 1 which is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$. Write an equation for a plane passing through the origin and spanned by the vectors $\mathbf{a}$ and $\mathbf{b}$.

4. Find the distance between the parallel planes $2x - 2y + z = 5$ and $2x - 2y + z = 20$.

5. Determine/describe/sketch level curves for the function $f(x, y) = 3x^2 + 2y^2$.

6. Determine whether the following limits exist. If they do, calculate the limit.

$$\lim_{(x,y)\to(1,2)} \frac{y^2}{x^2 + y^2}, \qquad \lim_{(x,y)\to(0,0)} \frac{y^2}{x^2 + y^2}$$

7. Calculate ∇f at the point $(1, 2)$ for $f(x, y)) = \ln(x^2 + y^2)$. Write an equation for the tangent plane to f at the point $(1, 2)$.

8. Compute the matrix Df where $f(x, y, z) = (xz - y^2, \sin x - \cos y)$.

9. Suppose that an insect flies along the helical curve

$$x = \cos(2\pi t), y = \sin(2\pi t), z = 3t,$$

and that the temperature (in some nice but nonstandard units) at any point (x, y, z) is given by the function

$$f(x, y, z) = (x^2 + y^2)z.$$

What is the instantaneous rate of change of temperature felt by the insect at time $t = \frac{1}{2}$? Also, what is the derivative of $f(x, y, z)$ in the direction $(0, -2\pi, \frac{1}{2})$?

10. Suppose a particle moving in 3-dimensional space with constant acceleration (due to gravity with a nicer constant)

$$\mathbf{a}(t) = (0, 0, -10).$$

Given an initial position $\mathbf{x_0} = (0, 0, 0)$ and initial velocity $\mathbf{v_0} = (2, 4, 10)$, what is the position function for the particle? Write the equation for the tangent line to the particle at $t = \frac{1}{2}$.