

Name:

ID:

Math 205: Final Exam

May 15, 2008

The exam is all partial credit. Please write neatly and clearly, showing all of your work. No calculators, cell phones, books, or notes may be used. The test contains 130 possible points, and you have 2.5 hours to work. Good luck!

| | |
|-------|--|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| Total | |

1. a. (5 points) Find a vector perpendicular to both $(1, 2, -1)$ and $(0, 1, 0)$.

b. (5 points) What is the angle between $\mathbf{i} + \mathbf{k}$ and $4\mathbf{i} + \mathbf{j} - \mathbf{k}$?

2. (7 points) Suppose a retail company's profit depends on 3 variables, x , y , and z . In turn, each of these 3 variables depends on the 2 variables u and t . For instance, let u be a measure of population density and t be time. Then, we could have $x(u, t)$ be a labor cost, $y(u, t)$ some economic indicator, and $z(u, t)$ a product cost.

Suppose that the profit is given by the function

$$P(x, y, z) = y^{\frac{1}{2}} z^{\frac{-1}{2}} - 2(x^2 + z).$$

Then, find the rate of change of profit with respect to time (i.e. find $\frac{\partial P}{\partial t}$). Your answer should be expressed in the variables x, y, z and the partial derivatives of x, y, z with respect to u, t .

3. (13 points total)

a. Find the equation of the tangent plane to

$$z = \cos x \sin y$$

at the point $(0, \frac{\pi}{3})$. (In case you forgot, $\cos \frac{\pi}{3} = \frac{1}{2}$, and $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$.)

b. Use linear approximation to estimate $\cos(.2) \sin(\frac{\pi}{3} - .1)$.

4. a. (10 points) Suppose a particle (in \mathbb{R}^2) moves subject to a conservative force with potential

$$f(x, y) = x^2 - xy + \frac{3}{2}y^2 + 5x + 4.$$

At what point(s) is the potential energy minimized?

b. (10 points) Suppose the particle from part (a) is constrained to live on the curve

$$x + 2y = 6.$$

Find any equilibrium points (i.e. critical points of the potential energy subject to our constraint).

5. (10 points) Calculate the following integral (Hint: change the order of integration)

$$\int_0^1 \int_{x^2}^1 x^3 \sin(y^3) dy dx.$$

6. (10 points) Let D be the parallelogram in \mathbb{R}^2 bounded by the four lines

$$x + y = 0, \quad x + y = \frac{\pi}{2}, \quad 2x - y = 0, \quad 2x - y = \pi.$$

Calculate the integral

$$\iint_D \cos(x + y) \sin(2x - y) dA.$$

(Hint: Use a change of variables.)

7. (5 points) Consider the integral

$$\iiint_E (x^2y + z) dV$$

where E is the region bounded by the planes

$$z = 2 - x - y, \quad x = 0, \quad y = 0, \quad z = 0.$$

Convert this integral into an iterated integral (i.e. properly set up bounds of integration for all three integrals). You do not need to evaluate.

8. Let $C : [0, 2\pi] \rightarrow \mathbb{R}^3$ be a curve in \mathbb{R}^3 given by

$$C(t) = (\cos t, \sin t, 2t).$$

a. (8 points) Find the length of the curve C .

b. (6 points) Suppose a force is given by the vector field

$$\mathbf{F} = (x, y, z).$$

What is the *average magnitude* of the force \mathbf{F} along the curve C ? You do not need to solve completely, but instead, you may stop once you have the answer expressed in terms of an ordinary integral.

8 (cont'd). We are still considering the curve C traced out by

$$C(t) = (\cos t, \sin t, 2t), \quad 0 \leq t \leq 2\pi$$

and the force

$$\mathbf{F} = (x, y, z).$$

c. (8 points) Calculate the work done by \mathbf{F} in transporting a particle along the curve C by computing the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{s}.$$

d. (8 points) Is \mathbf{F} conservative? If so, find a potential and use this to calculate part (c) in a different way.

9. a. (7 points) Show that

$$\int_C y \, dx + (2x + \sin(e^y)) \, dy = 0$$

for any curve C contained in the x -axis.

b. (8 points) Let γ be a curve in \mathbb{R}^2 given by

$$\gamma(t) = (\cos t, \sin t), \quad 0 \leq t \leq \pi.$$

Use part (a) to relate γ to a closed curve and then use **Green's Theorem** to calculate

$$\int_{\gamma} y \, dx + (2x + \sin(e^y)) \, dy.$$

10. (10 points) Let D be the triangle formed by the 3 points $(0, 0)$, $(1, 0)$, $(1, 2)$, and let C be the (counter-clockwise) curve along the boundary (tracing out the above triangle). Compute

$$\int_C (y + 1)e^{x^2} dx.$$

(Hint: Use Green's Theorem, and be careful about how you set up your new integral.)