

Name: _____ Recitation: _____ ID#: _____

MAT131: Midterm II

Solutions

Thursday, November 9 2006
8:30pm-10pm

Javits 100: LEC I and LEC III
Old Engineering 143: LEC II

Problems:	1	2	3	4	5	Total
Points:						

There are five problems. Do all work on these pages. **No** calculators, cell phones or notes may be used. The point value (out 100) of each problem is marked in the margin. Except for problems 1 and 4, you are expected to write the appropriate computations and justifications to get full credit. For each question, please put a rectangle around your final answer.

(10 pts) 1. For each of the question below, circle (T) if the statement is true and (F) if it is false. Each correct answer gives 1 point and each incorrect answer or unanswered question gives 0 point.

T F (a) A function may have a global maximum at three different numbers.

T F (b) If a function f is such that $f'(2) = 0$, then it necessarily has a local minimum or maximum at $x = 2$.

T F (c) $\lim_{h \rightarrow 0} \frac{2^h - 1}{h} = \log_2 e$.

T F (d) If f and g are two functions differentiable at $x = 15$, then their quotient $\frac{f}{g}$ is also differentiable at $x = 15$.

T F (e) $\frac{d^{31}}{dx^{31}} \cos x = \sin x$.

T F (f) $\frac{d}{dx} \arccos x = \frac{-1}{\sqrt{1-x^2}}$.

T F (g) $\frac{d}{dx} \ln(-e^x) = 1$.

T F (h) The linear approximation of $\sqrt{x+3}$ at $x = 6$ is

$$L(x) = 3 + \frac{x}{6}.$$

T F (i) If f is a continuous function on the interval $[0, 1]$, then f has a global minimum on this interval.

T F (j) If f and g are differentiable functions at $x = 3$, then their composite $f \circ g$ is differentiable at $x = 3$.

2. Compute the derivatives of the following functions.

(5pts) (a) $\frac{e^x \cos x}{x^3+1}$

(5pts) (b) $\sin(x^5 + 1)$

(5pts) (c) $x^4 - 23 \cos(3x) + \ln(x^2)$

(5pts) (d) $(2x)^{x \cos(x)}$

(5pts) (e) $\arctan(\ln x)$.

(5pts) (f) $\frac{e^{\sin x} (x^2+2)^5}{\sqrt{x}(\tan x)^2}$

3. Consider the parametric curve \mathcal{C} given by the equation

$$x(t) = e^{2t} - e^{-2t}, \quad y(t) = e^{2t} + e^{-2t}.$$

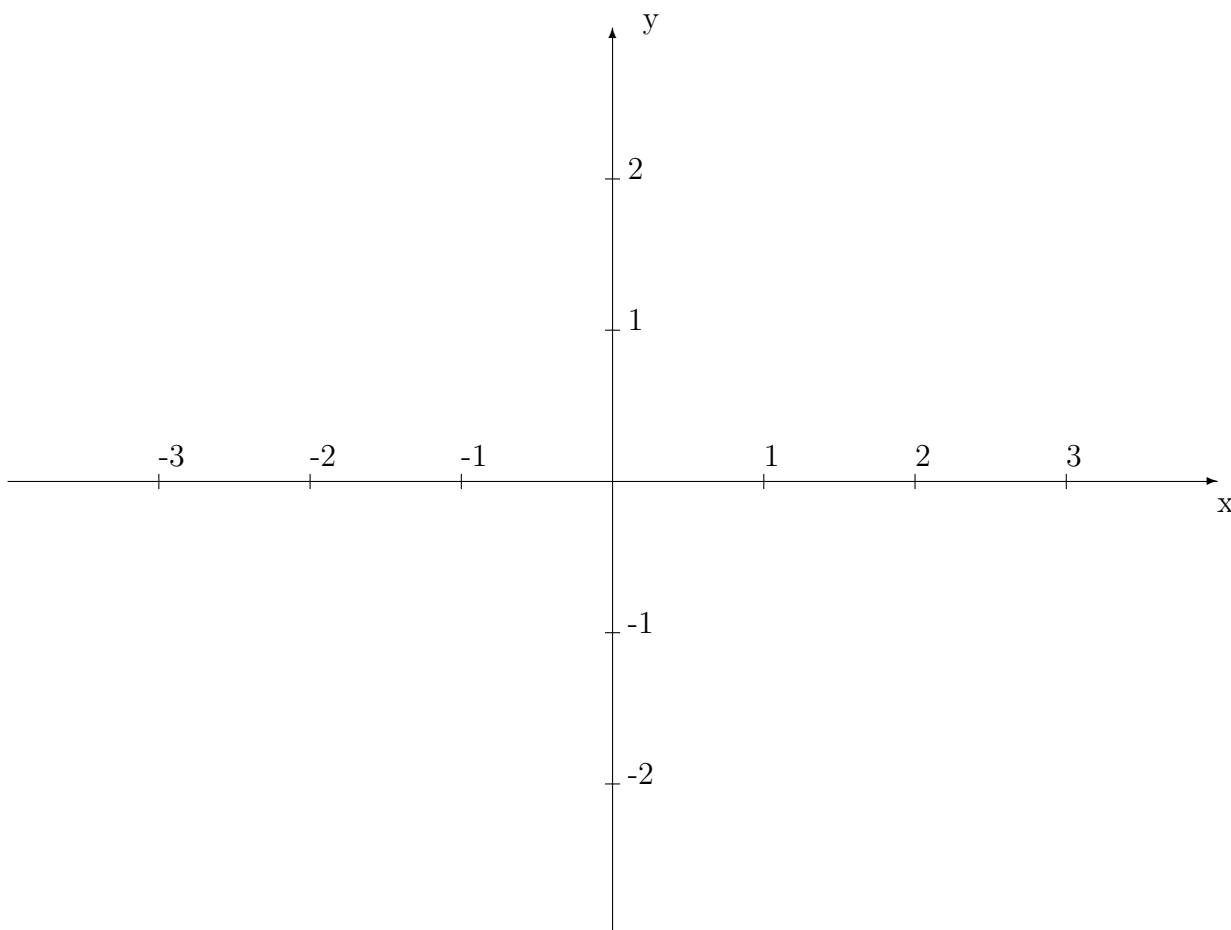
Alternatively, it is given implicitly by the equation: $x^2 - y^2 = -4$, $y > 0$.

(10pts) (a) Compute the slope $\frac{dy}{dx}$ of the tangent to the curve \mathcal{C} at the point $(x(t_0), y(t_0))$ for $t_0 \in \mathbb{R}$.

(10pts) (b) The curve \mathcal{C} intersects the ellipse \mathcal{E} given by the equation $\frac{x^2}{9} + \frac{y^2}{25} = \frac{1}{2}$ at the points $(-\frac{3}{2}, \frac{5}{2})$ and $(\frac{3}{2}, \frac{5}{2})$. By computing the slope of the tangent of \mathcal{E} at these points, check whether or not the curves \mathcal{C} and \mathcal{E} are orthogonal to each other.

(20pts) 4. Sketch the graph of a function f with domain $(-2, 2)$ satisfying the following conditions:

- i. The **linear approximation** of f at $x = 1$ is $L(x) = 2$;
- ii. $f'(x) > 0$ for $0 \leq x < 1$;
- iii. $f''(x) < 0$ for $0 < x < 2$;
- iv. $\lim_{x \rightarrow 2^-} f(x) = -\infty$;
- v. f is an odd function.



(20pts) **5.** Captain Nemo is in his motionless submarine (the Nautilus) 300 meters under the sea level waiting for his good old friend Captain Ahab to discuss about their misanthropic vision of the world over a cup of tea. Captain Ahab is in his whaling ship (the Pequod) moving at a constant speed toward the point of the water surface which is exactly above the Nautilus. Using his sonar, Captain Nemo is able to measure that the distance between the Nautilus and the Pequod is decreasing at a rate of $10m/s$ when this distance is 500 meters. What is the speed of the Pequod?

Recitations of MAT131

Recitation	Time	Location	TA
R01	TuTh 12:50pm- 1:45pm	Earth and Space 183	Disconzi, Marcelo
R02	MF 2:20pm- 3:15pm	Earth and Space 79	Cheng, Jonathan
R03	TuTh 8:20am- 9:15am	Earth and Space 181	Disconzi, Marcelo
R04	WF 11:45am-12:40pm	Earth and Space 181	Lehrer, Raquel
R05	MF 2:20pm- 3:15pm	Harriman Hll 115	Jaggi, Amit
R06	MW 10:40am-11:35am	Earth and Space 183	Weng, Luoying
R07	MW 6:50pm- 8:10pm	Earth and Space 183	Li, Yinghua
R08	MW 3:50pm- 4:45pm	Chemistry 128	Dutta, Satyaki
R09	TuTh 5:20pm- 6:15pm	Lgt Engr Lab 154	Hassan, Mohamed
R10	WF 9:35am-10:30am	Earth and Space 183	Weng, Luoying
R11	TuTh 3:50pm- 4:45pm	Lgt Engr Lab 154	Hassan, Mohamed
R12	TuTh 8:20am- 9:15am	Library N4000	Fong, Chee
R13	MF 12:50pm- 1:45pm	Library N4006	Li, Yinghua
R14	TuTh 3:50pm- 5:10pm	Physics P125	Fong, Chee