Name: ______________________ Recitation: __________ ID#: ______________

MAT131: Final Exam

Wednesday, December 20 2006
11:00am-1:30pm

Humanities 1003: R01, R03 and R05
Humanities 1006: R02 and R04
Harriman 137: LEC2
Old Chemistry 116: LEC3

Problems:	1	2	3	4	5	6	7	Total
Points:								

There are seven problems. Do all work on these pages. **No** calculators, cell phones or notes may be used. The point value (out 200) of each problem is marked in the margin. Except for problem 1, you are expected to write the appropriate computations and justifications to get full credit. For each question, please put a rectangle around your final answer.

(**10 pts**) **1.** For each of the question below, circle (**T**) if the statement is true and (**F**) if it is false. Each correct answer gives 1 point and each incorrect answer or unanswered question gives 0 point.

T **F** (a) If a function f has an antiderivative F, then F is necessarily continuous.

T **F** (b) If f is an even function on the interval $[-\pi, \pi]$, then necessarily

$$\int_{-\pi}^{\pi} f(x)dx = 0.$$

T **F** (c) $\displaystyle\sum_{i=1}^{20} i = 210.$

T **F** (d) The equation $x^2 + 2x + 1 = 0$ has a solution in the interval $[0, 1]$.

T **F** (e) If a function f is differentiable on an interval $[a, b]$, then f has a global minimum on $[a, b]$.

T **F** (f) If f and g are functions differentiable at $x = a$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}, \quad \text{provided } g'(a) \neq 0.$$

T **F** (g) If $f(x) > 1$ for all $x \in [0, 1]$, then

$$\int_{0}^{1} f(x)dx \geq 0.$$

T **F** (h) If f is concave up on an open interval (a, b), then f necessarily has a global minimum on (a, b).

T **F** (i) If $f(x) = (x^3 + 1)^3$, then there exists a number $c \in (0, 1)$ such that

$$f'(c) = 7.$$

T **F** (j) If $\lim_{x \to a} f(x) = 3$ and $\lim_{x \to a} g(x) = 2$, then

$$\lim_{x \to a} (f(x) + g(x)) = 6.$$

2. Compute the following derivatives.

(5pts) **(a)** $\frac{d}{dx}\frac{\cos(e^x+x^2)}{2x}$

(5pts) **(b)** $\frac{dy}{dx}$ if $2(x^2 + y^2)^2 = 25(x^2 - y^2)$

(5pts) **(c)** $\frac{d}{dx}(\ln x)^{3x}$

(5pts) **(d)** $\frac{d}{dx}\int_x^{x+2} t^2 e^{t^2}\,dt$

3. Consider the function $f(x) = xe^{-\frac{x^2}{2}}$.

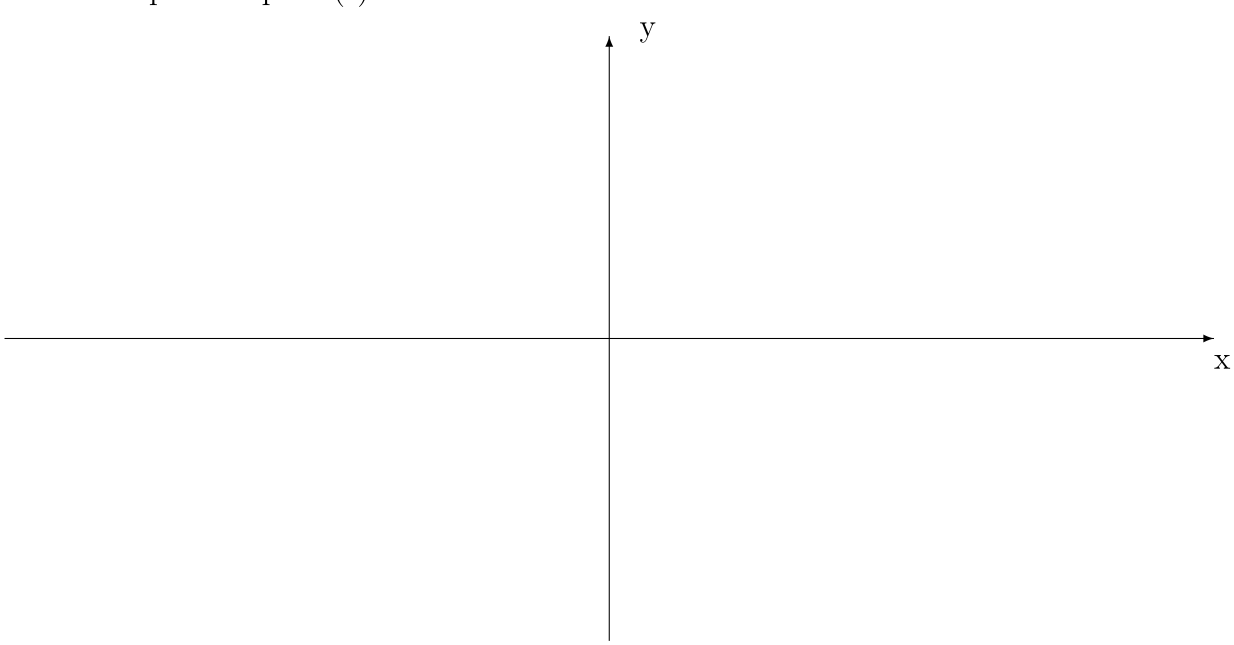
(5pts) **(a)** Is it an even function, an odd function or is it neither even nor odd?

(10pts) **(b)** Find the horizontal and vertical asymptote(s) of the function f.

(10pts) **(c)** Find the critical number(s), if any, and the intervals of increase and decrease of the function f.

(10pts) **(d)** Find the inflection point(s), if any, and the intervals of concavity of the function f.

(15pts) **(e)** From steps (a), (b), (c) and (d), draw the graph of the function f. Label all important point(s).



(40pts) **4.** A cheesemaker wants to make a cheese of volume $V = 6L$ (6 liters correspond to $6000cm^3$) shaped like a circular cylinder. This cheese will be cut into six equal pieces, each piece being obtained by cutting a sector of the cylinder of angle $\frac{\pi}{3}$. The cheesemaker would like to wrap each piece in plastic foil. What are the radius and the height of the cylinder that will minimize the amount of plastic foil needed?

5.

(10pts) **(a)** Use the linear approximation of the function $f(x) = \sqrt[3]{x}$ at $x = 1$ to get an approximation of the number $\frac{1}{\sqrt[3]{2}} = \sqrt[3]{\frac{1}{2}}$.

(10pts) **(b)** Use Newton's method with the equation $\frac{1}{x^3} - 2 = 0$ and starting with $x_1 = 1$ to find the third approximation x_3 of the number $\frac{1}{\sqrt[3]{2}}$.

(5pts) **(c)** Is the approximation in (b) bigger or smaller than the actual value of $\frac{1}{\sqrt[3]{2}}$?

6. Compute the following limits.

(5pts) **(a)** $\lim\limits_{x\to 0^-} \frac{\cos^2 x - 1}{\sin(2x)}$

(5pts) **(b)** $\lim\limits_{x\to +\infty} \frac{e^{-x}}{x}$

(5pts) **(c)** $\lim\limits_{n\to\infty} \left(\sum_{i=1}^{n} \frac{3}{n} \left(\frac{3i}{n} + 1 \right)^3 \right)$

(5pts) **(d)** $\lim_{x \to 0^+} (\tan x)^x$

(5pts) **(e)** $\lim_{x \to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$

7. Compute the following integrals.

(5pts) **(a)** $\int \left(\cos(x) + \sec^2(x)\right) dx$

(5pts) (b) $\int \frac{(\ln x)^2}{x} dx$

(5pts) (c) $\int \frac{\arcsin x}{\sqrt{1-x^2}} dx$

(5pts) **(d)** $\int_1^3 (x-2)(x+3)dx$

(5pts) **(e)** $\int_0^{\frac{\pi}{4}} (\sec^2 x)(\tan x)dx$

(5pts) (f) $\int_{-3}^3 \sqrt{1-\frac{x^2}{9}}dx$

Recitations of MAT131

Recitation	Time	Location	TA
R01	TuTh 12:50pm- 1:45pm	Earth and Space 183	Disconzi, Marcelo
R02	MF 2:20pm- 3:15pm	Earth and Space 79	Cheng, Jonathan
R03	TuTh 8:20am- 9:15am	Earth and Space 181	Disconzi, Marcelo
R04	WF 11:45am-12:40pm	Earth and Space 181	Lehrer, Raquel
R05	MF 2:20pm- 3:15pm	Harriman Hll 115	Jaggi, Amit
R06	MW 10:40am-11:35am	Earth and Space 183	Weng, Luoying
R07	MW 6:50pm- 8:10pm	Earth and Space 183	Li, Yinghua
R08	MW 3:50pm- 4:45pm	Chemistry 128	Dutta, Satyaki
R09	TuTh 5:20pm- 6:15pm	Lgt Engr Lab 154	Hassan, Mohamed
R10	WF 9:35am-10:30am	Earth and Space 183	Weng, Luoying
R11	TuTh 3:50pm- 4:45pm	Lgt Engr Lab 154	Hassan, Mohamed
R12	TuTh 8:20am- 9:15am	Library N4000	Fong, Chee
R13	MF 12:50pm- 1:45pm	Library N4006	Li, Yinghua
R14	TuTh 3:50pm- 5:10pm	Physics P125	Fong, Chee