MATH 320, FALL 2017 PRACTICE MIDTERM 2

NOVEMBER 7

Each problem is worth 10 points.

Problem 1.

a. (4 points) State the Intermediate Value Theorem.

b. (6 points) Prove that a continuous function $f:[0,1]\to [0,1]$ has a fixed point x, satisfying f(x)=x.

Problem 2.

a. (3 points) Let f be a real valued function on a metric space (S, d). State one of the two equivalent definitions of continuity of f at a point $x \in S$.

b. (7 points) Prove that a continuous function on a closed bounded interval [a, b] is bounded.

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Problem 3. (10 points) State the alternating series test. Using this, or otherwise, prove that the limit

$$\lim_{N \to \infty} \left(\sum_{n=1}^{N} \frac{1}{n} - \log N \right)$$

exists and is finite. (Remark: this number is called Euler's constant.)

Problem 4.

a. (3 points) State the definition of a countable set S.

b. (7 points) Prove that the set of sequences $\{a_n\}_{n\in\mathbb{N}}$ with values in $\{0,1\}$ is uncountable.