MATH 307, FALL 2020 PRACTICE MIDTERM 1

SEPTEMBER 28

Each problem is worth 10 points.

Problem 1. Determine the distance between
$$\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$$
 and the line $\ell(t) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} +$

$$t \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$
.

Problem 2. Express $u = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ as a component parallel to $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and a component perpendicular.

Problem 3. For each of the linear systems below, express the system in row-reduced form and give the solution set.

a.

$$x + 2z = 7$$
$$2x + y + z = 3$$
$$y + 2z = 1.$$

b.

$$x + y + z = 10$$
$$2x + y = 7.$$

Problem 4.

a. Calculate the inverse matrix of

$$\begin{pmatrix} 3 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

b. Calculate the determinant

$$\det \begin{pmatrix} 3 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Problem 5. Find the equation of the plane through

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}.$$

Problem 6. Find the equation of the plane tangent to $2x^2 + 2y^2 - z^2 = 12$ at $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$.

at
$$\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$
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