Teaching Statement

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In second grade, when we were learning multiplication, my teacher was concerned that I had a learning disability. If given the problem, say, 5 times 7, I would first count by 5s up to 35, then count by 7s back down to 0 to double check my answer. All of the other students, apparently, were memorizing multiplication tables; the result was that I was by far the slowest in the class. My mother, at the teacher's behest, bought me flash cards and tried to make me memorize tables too. I hated it, and for the next several years math was something that was easy but boring. In ninth grade, my geometry teacher changed everything. We were given no textbooks, just blank paper and a set of postulates. At first things were easy and obvious — proving that Socrates was mortal seemed like a waste of breath — but gradually the beauty, power, and elegance, the *correctness*, of what we were doing became apparent. That class wasn't about memorizing and correctly applying formulae, or reciting terminology, or even getting the correct answer. We weren't being taught facts; we were being taught a powerful, precise way to think.

I want all math students to experience the kind of revelation I experienced in that geometry class. Furthermore, I don't think this is an unrealistic goal. However, most math education apparently fails to encourage mathematical thought. Far too many of the students I have encountered in my teaching career have been trained to think of mathematics as an abstract shell game of meaningless symbols and arbitrary rules. Such an understanding is like an epistomological thought experiment made nightmarishly real — it mimics real understanding well enough to allow the student to pass a math class, while imparting nothing of substance. It is important for students to realize that what they are being taught is essentially a new way of thinking about things. This is my primary goal in teaching mathematics: to get the students to think mathematically, to understand what is going on on a meaningful, intuitive level, rather than blindly copying what is done in class or in the book or on the internet.

Unfortunately, convincing students to connect mathematical symbols with actual meaning can sometimes entail a struggle against years of training. In my office hours last year, I was leading a student through a problem, and he arrived at the answer 8/2. When I told him to simplify, he reached for his calculator. I stopped him, and he was stumped; I had to talk him through the process of division as well. The problem was that the numbers 8 and 2 themselves were not meaningful — they were just abstract symbols to be manipulated. "Suppose we have 8 dollars..." He was delighted when he realized that numbers were "just like money", and building on that we worked back up until he understood what was actually happening when he manipulated an equation, and why he had to do the same thing to both sides. All of a sudden, what we had been doing in class for weeks made *sense*.

Connecting meaning and mathematics can also sometimes entail a struggle against the way things are typically taught, or against the way that students want to learn. Students may sometimes say, "Just show me how to do it," and some textbooks accomodate them. At Stony Brook, linear algebra in particular seems to be fraught with examples of students learning an algorithm for performing some computation without the faintest idea of what they're doing or why. As a graduate student at Stony Brook, I spend several hours a week tutoring undergraduates in the math learning center, and there are frequently linear algebra students who are confused about some abstract set of rules they're trying to follow to solve a problem. When I help these students, I always talk the problem through from the beginning, essentially forcing the student to rederive the theorem or algorithm she is supposed to be using from scratch. We'll work a few examples like that, and invariably the student will be amazed at how simple the problem really was, even though I've made her do more work. It's like turning on a light; because she has figured out the solutions to these problems on her own, the algebraic processes become meaningful, and the student becomes capable of answering her own questions and addressing her own hesitations.

I believe that the foundation of mathematical thinking is understanding that mathematics is a series of obvious steps that takes one from an obvious (or assumed) premise to a non-obvious conclusion. I always explain this to my students, and whatever the class is I try to teach it in such a way that all the students can see this structure, this gradual crescendo of obvious steps. Students are frequently tempted to ignore the background and meaning behind the problem, because they just want to get an answer; a big part of the reason for this, I think, is a misapprehension that the real story is too difficult for them to understand. To convince the students that the steps are in fact obvious, I frequently rely on Socratic dialogue even in lecture, asking leading questions but forcing the class to come up with the answers. When teaching a small number of students, I use Socratic dialogue almost invariably. I strive to keep classes quite informal and to remain as approachable as possible, so that my students will be willing to ask a question any time they don't understand one of the steps.

An important part of exploring math though dialogue is the active elimination of incorrect answers. I like to explore possible incorrect answers; again, the idea here is to convince the students that they can recognize these answers as incorrect for themselves. If possible, I like to get the students to find the flaws in their answers themselves. With more complicated ideas, I will try to make a game of it, defeating idea after idea with counterexamples until the class arrives at the right answer. If the students can see why the correct answer is correct, and why the incorrect answers are incorrect, then the rules they are learning cease to be arbitrary. If they further realize that they theselves have the ability to distinguish correct from incorrect answers, then those rules lose their mystique, and the students become more willing to try actual thought in place of the slavish application of memorized algorithms.

I believe that almost everyone, if they can overcome their fear and awe and and think through the steps of problems using everyday logic and common sense, is capable of doing mathematics at a much higher level than the average US college graduate; it is a tragedy that the very power offered by clear mathematical thinking convinces people that they are incapable of it. More important than improved performance, though, is the lesson that the student is learning. For the vast majority of students, the specific calculation techniques learned in any given math class may have little bearing on their lives. If they can be taught to think mathematically, though, that is a crucially important lesson that will serve them well in any walk of life.