

MAT 320 Introduction to Analysis

Review Sheet for Midterm 2

The second midterm covers everything we learned since the first midterm; an outline of the material is found below. (You are also responsible for the material covered before the first midterm, but those topics will not be emphasized.) You must know all the definitions and statements of the important theorems, and also understand all proofs (although you will not be asked to reproduce a proof of a theorem presented in class). You can use (with clear references) all theorems and facts covered in class, but you have to explain all the steps in your proofs.

No books, calculators, or notes will be permitted.

Sequences

- convergence of a sequence
- convergent sequences are bounded
- alg. operations $+$, $-$, \times , \div on convergent sequences respect limits
- Squeeze Theorem
- subsequences
- a bounded sequence has a convergent subsequence (Bolzano-Weierstrass Theorem)
- (x_n) converges to L iff every subsequence of (x_n) converges to L
- Cauchy sequences
- a sequence in \mathbb{R} is convergent iff it is Cauchy
- convergence of infinite series

Cauchy sequences and infinite series will not be emphasized, but you can expect some short questions about these topics.

Functions and Continuity

- limit of a function at a point
- sequential criterion for the existence of limit
- algebraic operations on functions respect limits
- f is continuous at p iff $\lim_{n \rightarrow \infty} f(x_n) = f(p)$ for every sequence (x_n) in the domain of f that converges to p
- algebraic operations on functions respect continuity at a point
- a composition of two continuous functions is continuous
- a continuous function on $[a, b]$ is bounded (Boundedness Theorem)
- a continuous function on $[a, b]$ attains its maximum and minimum (Maximum-Minimum Theorem)
- continuous functions have the intermediate value property
- the inverse of a continuous monotone function $f : [a, b] \rightarrow \mathbb{R}$ is continuous
- uniform continuity
- a continuous function on a closed, bounded interval is uniformly continuous