MATH 320 HOMEWORK 1, DUE ON FRIDAY OCT 24

1. For each of the following sequences, find out whether it is convergent. If convergent, find the limit,

(1) $\frac{\sin n^2}{n}$, $\frac{n-2\cos n}{n+1}$
(2) $(\sqrt{n})^{1/2n}$, $(n+1)^{1/\ln(n+1)}$,
(3) $(1-\frac{1}{n})^n$, $\frac{n!}{n^n}$

2 Use the squeeze theorem to determine the limits of the following

$$n^{1/n^2}, \qquad (n!)^{1/n^2}$$

3 Let $p > 0$ and $a_1 > 0$, define $a_{n+1} = \sqrt{p + a_n}$. Show that (a_n) converges and find the limit.

4 Let $I_n = [a_n, b_n]$ be a sequence of nested intervals such that $\lim(b_n - a_n) = 0$. Show that $\lim a_n = \lim b_n$, and denoting the common limit by a, then $\bigcap_{n=1}^{\infty} I_n = a$.

5 (1)Suppose (a_n) is a sequence with positive terms $a_n > 0, \forall n$. Also for some $N \in \mathbb{N}$ the following inequality holds

$$\frac{a_{n+1}}{a_n} \leq 1 - \frac{1}{n} \qquad \forall n > N. \tag{0.1}$$

Prove $\lim a_n = 0$.

(2)Is (a_n) convergent, if we modify (0.1) to be

$$\frac{a_{n+1}}{a_n} \leq 1 - \frac{1}{2n} \qquad \forall n > N. \tag{0.2}$$

If convergent, find the limit.

(3) Find $\lim \frac{1}{n!}(\frac{n}{e})^n$.

6 Establish the convergence or the divergence the sequence

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots \frac{1}{2n}, \quad \forall n \in \mathbb{N}.$$