Review Sheet for Final Exam

Mathematics 131

May 15, 2006

The exam is cumulative. It will cover all sections on which homework was assigned: Chapter 1 (except 1.4 and 1.7), Chapter 2, Chapter 3 (except 3.3), Chapter 4 (except 4.4 and 4.7), and Sections 5.1–5.5 of Chapter 5.

No calculators of any kind will be allowed.

For the final exam, you will be allowed a single side of a single 8.5 x 11 inch piece of paper to use as a formula sheet. The sheet must be handwritten and legible to the slightly-near-sighted human eye. You should bring it with you to the final exam, and you will have to staple it to the back of your exam. I will deduct at least 10% of your final exam grade for formula sheets that don't comply with these rules.

Be able to:

- Use a linear approximation to estimate things like $\sqrt{8.5}$, or to estimate the error in volume of a sphere when the radius is measured with error Δr.
- Compute the rate at which one function of time is changing (e.g., the hypotenuse of a triangle), in terms of the rate of another function (e.g., one side of the triangle).
- Find the absolute maximum and minimum values of a continuous function on a closed interval, using the critical point/end point method. Understand why this may be impossible if the function is not continuous or if the interval is not closed.
- Identify local maxima, local minima, and inflection points (and understand the distinction between $f'(x) = 0$ and a local extreme, and between $f''(x) = 0$ and an inflection point!). Be able to determine intervals on which a function is increasing or decreasing, and when it is concave up or down, and use this information to get an accurate graph.
- Compute limits of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ using L'Hopital's rule. Know how to change other forms, like $0 \cdot \infty$, $\infty - \infty$, 0^0, 1^∞, and ∞^0, into the standard form. Remember only to use L'Hopital's rule when the limit actually is indeterminate!
- Solve optimization word problems, setting up diagrams and relating variables. Remember to always determine your endpoints!
- Set up the iteration for Newton's method and go through a few steps by hand. Also understand what Newton's method means graphically.

- Graph antiderivatives of functions, either roughly (using only local maxima/minima and concavity) or precisely (using estimates of areas).
- Compute by hand a small left- or right-hand sum for a particular area. Know what these sums mean graphically (i.e., be able to draw the rectangles). Understand the simple formula for $R_n - L_n$ and how it can be used to estimate errors.
- Compute integrals of polynomials (up to cubic) directly from the definition (as the limit of right-hand sums as the number of divisions approaches infinity). Be able to set up a general integral-from-the-definition. Understand sigma notation!
- Evaluate definite integrals of basic functions.
- Find distance travelled in terms of velocity (in particular, be able to integrate the absolute value!). Understand the connection between distance, speed, position, acceleration, and velocity.
- Use the Fundamental Theorem of Calculus to understand functions like "erf" that are defined by integrals. Be able to compute derivatives of things like $x \int_0^x f(t)\,dt$ or $\int_x^{2x} f(t)\,dt$.
- Use the substitution rule to evaluate indefinite or definite integrals.

Review problems:

Make sure you know *exactly* how to do all of these. It's not enough to skim through the list! If you haven't tried all of these problems, and you do badly on the exam, *it's your own fault.*

If you get stuck with any of them, go back and do some extra exercises from the corresponding section until you understand.

- Chapter 3 review, pp. 255–257: (Exercises): 71a, 72.
- Chapter 4 review, pp. 335–338: (Concept check): 1–2, 4–10; (True-false): 1–13; (Exercises): 1–12, 23, 25–32, 34–35, 37–38, 42, 44, 47–48 (for 47–48, just set up the Newton's method iteration), 49–54, 56.
- Chapter 5 review, pp. 433–436: (Concept check): 2, 5–7, 13; (True-false): 1–10, 13–15; (Exercises): 1–5, 7, 8, 9–20, 25–26, 28, 39–42, 47–48 (use left/right estimates instead, for $n = 4$; set up the sums; you don't need to calculate them).