MAT 131 HW solutions (5.1–5.2)

1 Section 5.1

1. (a) $L_5 = 2(1 + 3 + 4.4 + 5.4 + 6.2) = 40$ and $R_5 = 2(3 + 4.4 + 5.4 + 6.2 + 7) = 52$.

   (b) $L_{10} = 1(1+2.2+3+3.7+4.4+4.8+5.3+5.8+6.3+6.6) = 43.1$ and $R_{10} = 1(2.2+3+3.7+4.4+4.8+5.3+5.8+6.3+6.6+7) = 49.1$.

4. (a) $R_5 = 1(24+21+16+9+0) = 70$. This is an underestimate, since the graph is decreasing.

   (b) $L_5 = 1(25+24+21+16+9) = 95$. This is an overestimate, since the graph is decreasing.

11. $L_6 = 0.5(0 + 6.2 + 10.8 + 14.9 + 18.1 + 19.4) = 34.7$ feet. $R_6 = 0.5(6.2 + 10.8 + 14.9 + 18.1 + 19.4 + 20.2) = 44.8$ feet. Since the speed was increasing, the actual distance (the integral) must be between these two numbers.

16. We'll use an average of left-hand and right-hand sums. The left sum is $L_6 = (5 \text{ sec})(0 + 50 + 79 + 88 + 95 + 98 \text{ km/h}) = 2050 \text{ km} \cdot \text{sec/h}$. Of course, the units are screwed up; we have to use the fact that 1 h = 3600 sec to get $L_6 = \frac{2050}{3600} \approx 0.57$ kilometers, or 570 meters.

    The right sum is similarly $R_6 = 5(50 + 79 + 88 + 95 + 98 + 100) = 2550 \text{ km} \cdot \text{sec/h} = \frac{2550}{3600}$ km, so $R_6 \approx 0.71$ kilometers, or 710 meters.

    The average of these two is a good estimate for the area; it's 640 meters.

17. The area is

$$\lim_{n\to\infty} \frac{15}{n} \sum_{i=1}^{n} \left(1 + \frac{15i}{n}\right)^{1/4}.$$

20. (a) The area under $y = x^3$ from $x = 0$ to $x = 1$ is

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 = \lim_{n\to\infty} \frac{1}{n^4} \sum_{i=1}^{n} i^3.$$

   (b) The area is

$$\lim_{n\to\infty} \frac{1}{n^4} \frac{n^2(n+1)^2}{4} = \frac{1}{4}.$$

2 Section 5.2

1. $R_4 = \frac{1}{2}\big[1.75 + 1 + -0.25 + -2\big] = 0.25.$

6. (a) $R_6 = 1(1 + -0.5 + -1.5 + -1.5 + -0.5 + 2.5) = -0.5$
   (b) $L_6 = 1(2 + 1 + -0.5 + -1.5 + -1.5 + -0.5) = -1.0$
   (c) $M_6 = 1(1.7 + 0 + -1 + -1.75 + -1.1 + 0.5) = -1.65$

7. $L_5 = 5(-42 - 37 - 25 - 6 + 15) = -475$ and $R_5 = 5(-37 - 25 - 6 + 15 + 36) = -85$. Since f is increasing, we know
$$-475 < \int_0^{25} f(x)\,dx < -85$$

9. $M_4 = 2[f(3) + f(5) + f(7) + f(9)] = 2(\sqrt{28} + \sqrt{126} + \sqrt{344} + \sqrt{730}) = 124.1644.$

12. $M_4 = 1[f(1.5) + f(2.5) + f(3.5) + f(4.5)] = (1.5)^2e^{-1.5} + (2.5)^2e^{-2.5} + (3.5)^2e^{-3.5} + (4.5)^2e^{-4.5} = 1.6099$

17. $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \sin x_i \Delta x = \int_0^{\pi} x \sin x\,dx.$

21. $\int_{-1}^{5} (1 + 3x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} (1 + 3x_i)\Delta x.$

    Since $\Delta x = \frac{5-(-1)}{n} = \frac{6}{n}$ and $x_i = -1 + i\Delta x = -1 + \frac{6i}{n}$, we get
$$\begin{aligned}
\int_{-1}^{5} (1 + 3x)\,dx &= \lim_{n\to\infty} \frac{6}{n} \sum_{i=1}^{n} \Big(1 + 3\big(-1 + \frac{6i}{n}\big)\Big) \\
&= \lim_{n\to\infty} \frac{6}{n} \sum_{i=1}^{n} (-2 + \frac{18}{n} i) \\
&= \lim_{n\to\infty} \frac{6}{n} \left(-2n + \frac{18}{n}\frac{n(n+1)}{2}\right) \\
&= \lim_{n\to\infty} \left(-12 + \frac{54(n+1)}{n}\right) \\
&= -12 + 54 \\
&= 42
\end{aligned}$$

24. $\displaystyle\int_0^5 (1+2x^3)\,dx = \lim_{n\to\infty}\sum_{i=1}^n (1+2x_i^3)\Delta x.$

Here $\Delta x = \frac{5}{n}$ and $x_i = 0 + i\Delta x = \frac{5i}{n}$, so that

$$\begin{aligned}\int_0^5 (1+2x^3)\,dx &= \lim_{n\to\infty}\frac{5}{n}\sum_{i=1}^n (1+2(\tfrac{5i}{n})^3)\\ &= \lim_{n\to\infty}\frac{5}{n}\sum_{i=1}^n (1+\tfrac{250}{n^3}i^3)\\ &= \lim_{n\to\infty}\frac{5}{n}\left(n+\frac{250}{n^3}\frac{n^2(n+1)^2}{4}\right)\\ &= \lim_{n\to\infty}\left(5+\frac{625}{2}\frac{(n+1)^2}{n^2}\right)\\ &= 317.5\end{aligned}$$

31. (a) $\displaystyle\int_0^2 f(x)\,dx = 4$

(b) $\displaystyle\int_0^5 f(x)\,dx = 10$

(c) $\displaystyle\int_5^7 f(x)\,dx = -3$

(d) $\displaystyle\int_0^9 f(x)\,dx = 2$

35. $\displaystyle\int_{-3}^0 (1+\sqrt{9-x^2}) = 3 + \frac{9}{4}\pi$ (a rectangle of width 3 and height 1, plus a quarter-circle of radius 3).

38. $\displaystyle\int_0^{10} |x-5|\,dx = 25$ (two triangles each of base 5 and height 5).

42. $\displaystyle\int_1^4 f(x)\,dx = \int_1^5 f(x)\,dx - \int_4^5 f(x)\,dx = 12 - 3.6 = 8.4$

45. $\displaystyle\int_0^1 (5-6x^2)\,dx = 5\int_0^1 dx - 6\int_0^1 x^2\,dx = 5\cdot 1 - 6\cdot\frac{1}{3} = 3$