# MAT 131 HW solutions (3.8–4.3)

## 1 Section 3.8

4. $N(1984) \approx 17.0 + \frac{2.0}{5}(-1) = 16.6$. $N(2006) \approx 24.9 + \frac{2.9}{5}(6) = 27.3$.

8. $f(x) = x^{3/4}$, so $f'(x) = \frac{3}{4}x^{-1/4}$. Thus $f(16) = 8$ and $f'(16) = \frac{3}{8}$, so $L(x) = 8 + \frac{3}{8}(x - 16)$.

15. $f(x) = x^5$, so $f'(x) = 5x^4$. The linearization at $a = 2$ is $L(x) = 32 + 80(x - 2)$, so
$$f(2.001) \approx L(2.001) = 32 + 80 \cdot 0.001 = 32.08.$$

18. $f(x) = \frac{1}{x}$, so $f'(x) = -x^{-2}$. The linearization at $a = 1000$ is $L(x) = 10^{-3} - 10^{-6}(x - 1000)$, so that $f(1002) \approx L(1002) = 10^{-3} - 10^{-6}(2) = 0.000998$.

33. (a) From the graph, $f'(1) = 2$, so the linearization is $L(x) = 5 + 2(x - 1)$. Thus $f(0.9) \approx 4.8$ and $f(1.1) \approx 5.2$.
    (b) Since $f'$ is decreasing, $f$ is concave down. For a concave down function, the tangent line is always above the actual function. Thus the values obtained by the linearization are too large.

## 2 Section 4.1

4. Since $A = LW$, we have $\frac{dA}{dt} = L\frac{dW}{dt} + W\frac{dL}{dt}$, and plugging in, we get $\frac{dA}{dt} = 20 \cdot 3 + 10 \cdot 8 = 140$ cm$^2$/s.

9. The surface area of a sphere is $S = 4\pi r^2$, where $r$ is the radius of the sphere. Since the diameter is $d = 2r$, we have $S = \pi d^2$, so that $S'(t) = 2\pi d(t)d'(t)$. We are given $S'(t) = 1$ cm$^2$/min and $d(t) = 10$ cm, so that $d'(t) = \frac{1}{20\pi}$ cm/min.

13. If the cars both start at the origin, the southbound car S is at point $(0, -60t)$ after $t$ hours while the westbound car W is at point $(-25t, 0)$ after $t$ hours. So the distance between them is $D(t) = t\sqrt{60^2 + 25^2} = 65t$. Thus $D'(t) = 65$ for any $t$, in particular at $t = 2$.

16. When the runner is $x$ feet away from home plate, heading towards first, the distance to second base, by the Pythagorean theorem, is $D = \sqrt{(90-x)^2 + 90^2}$. The time derivative is

$$D'(t) = \frac{1}{2}((90-x)^2 + 90^2)^{-1/2}\frac{d}{dt}[(90-x)^2 + 90^2] = \frac{-(90-x)x'(t)}{\sqrt{(90-x)^2 + 90^2}}.$$

When $x(t) = 45$ and $x'(t) = 24$, we have

$$D'(t) = \frac{-45 \cdot 24}{\sqrt{45^2 + 90^2}} = -10.7 \text{ ft/s}.$$

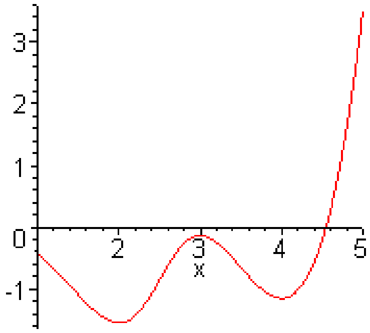
# 3 Section 4.2

1. An absolute maximum is a point $c$ such that $f(x) \leq f(c)$ for *every* $x$ in the domain of $f$. A local maximum is a point $c$ such that $f(x) \leq f(c)$ for *nearby* $x$ (on both sides). A point may be an absolute maximum but not a local maximum (if the maximum occurs at an endpoint of a closed interval), and it may be a local maximum but not an absolute maximum (if there is another point in the interval with larger value).

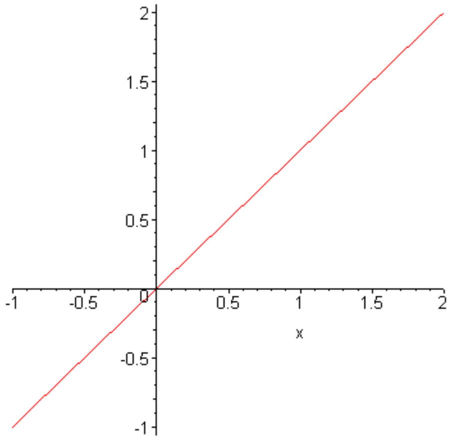
3. $a$ is neither; $b$ is a local maximum and an absolute maximum; $c$ is neither; $d$ is a local minimum and an absolute minimum; $e$ is a local maximum; $r$ is neither; $s$ is a local minimum; $t$ is neither.

6. The absolute maximum value is $f(8) = 5$. The absolute minimum value is $f(2) = 0$. The local maximum values are $f(1) = 2$, $f(4) = 4$, and $f(6) = 3$. The local minimum values are $f(2) = 0$, $f(5) = 2$, and $f(7) = 1$. (Since we are looking for maximum/minimum *values*, we give the $y$-values rather than the $x$-values as the answers.)

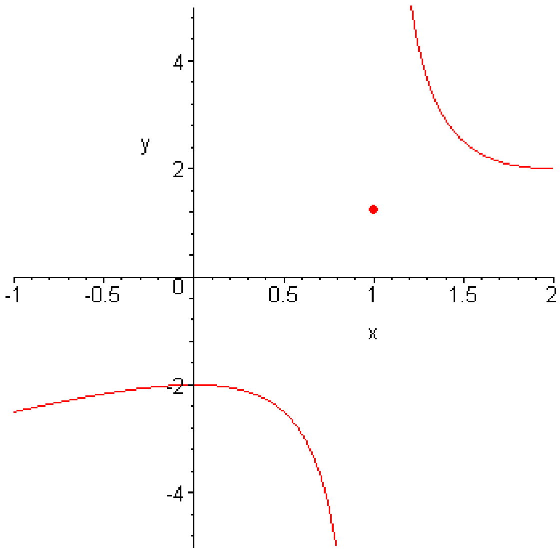
9. Here is one possible graph.



12. (a) Any example for which the absolute maximum occurs at an endpoint and the function never changes direction will do. Here's one.



(b) A continuous function on a closed interval $[-1, 2]$ *must* have an absolute maximum. Thus the only way to get a function with no absolute maximum is to have it be discontinuous somewhere. Here we show the graph of a function with a vertical asymptote at $x = 1$ (we define $f(1)$ arbitrarily, just to ensure that $f$ is actually defined everywhere) and a local maximum at $x = 0$.



23. $f(x) = 5x^2 + 4x$, so $f'(x) = 10x + 4$, and setting $f'(x) = 0$ we get the critical number $x = -\frac{2}{5}$.

28. $g(t) = |3t - 4|$, so

$$g'(t) = \begin{cases} 3 & t > \frac{4}{3} \\ -3 & t < \frac{4}{3} \end{cases}$$

So $g'(t)$ is never zero; however $g'(\frac{4}{3})$ does not exist, so $t = \frac{4}{3}$ is the only critical number.

37. $f(x) = 3x^2 - 12x + 5$ on $[0, 3]$. The critical numbers are found from $f'(x) = 6x - 12 = 0$, so we get $x = 2$, which is in the interval $[0, 3]$. Thus our list is

| 0 | 2 | 3 |
|---|---|---|
| $f(0) = 5$ | $f(2) = -7$ | $f(3) = -4$ |

Therefore the absolute maximum value is 5 and the absolute minimum value is $-7$.

40. $f(x) = x^3 - 6x^2 + 9x + 2$ on $[-1, 4]$. The critical numbers are found from $f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x - 1)(x - 3) = 0$, so $x = 1$ and $x = 3$. These are both in the interval $[-1, 4]$. So our list is

| $-1$ | $1$ | $3$ | $4$ |
|---|---|---|---|
| $f(-1) = -14$ | $f(1) = 6$ | $f(3) = 2$ | $f(4) = 6$ |

Therefore the absolute maximum value is 6 and the absolute minimum value is $-14$.

45. $f(x) = \sin x + \cos x$ on $[0, \pi/3]$. The critical numbers are found from $f'(x) = \cos x - \sin x = 0$, so that $\tan x = 1$. The only solution of this in $[0, \pi/3]$ is $x = \pi/4$. So our list is

| $0$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ |
|---|---|---|
| $f(0) = 1$ | $f(\frac{\pi}{4}) = \sqrt{2}$ | $f(\frac{\pi}{3}) = \frac{1+\sqrt{3}}{2}$ |

Therefore the absolute maximum value is $\sqrt{2}$ and the absolute minimum value is 1.

48. $f(x) = x - \ln x$ on $[\frac{1}{2}, 2]$. The critical numbers are found from $f'(x) = 1 - \frac{1}{x}$, whose only solution is $x = 1$. Thus our list is

| $\frac{1}{2}$ | $1$ | $2$ |
|---|---|---|
| $f(\frac{1}{2}) = \frac{1}{2} + \ln 2$ | $f(1) = 1$ | $f(2) = 2 - \ln 2$ |

Thus the absolute maximum value is $2 - \ln 2$ and the absolute minimum value is 1.

60. (a) $f(x) = ax^3 + bx^2 + cx + d$. To find the critical numbers, we solve $f'(x) = 3ax^2 + 2bx + c = 0$. This is a quadratic equation, which has solutions

$$x = \frac{-2b \pm \sqrt{4b^2 - 12ac}}{6a} = \frac{-b \pm \sqrt{b^2 - 3ac}}{3a}.$$

Depending on whether $b^2 - 3ac$ is positive, zero, or negative, we may have two, one, or no solutions. For example, $f(x) = x^3 + x^2$ has two critical points, $f(x) = x^3 + 3x^2 + 3x$ has one critical point, and $f(x) = x^3 + x$ has no critical points.

(b) If there are two critical points, then there are two local extreme values: one local maximum and one local minimum. If there is one critical point, then it must be a stationary point, so there are no local extrema. If there are no critical points, then there are again no local extrema.

# 4 Section 4.3

5. The inflection points are $x = 1$ and $x = 7$, when $f''$ changes sign. $x = 4$ looks like it might be an inflection point, but $f''$ does not change sign there.

6. (a) $f$ is increasing when $f'$ is positive, which happens on $[2, 4]$ and $[6, 9]$.

   (b) $f$ has a local maximum when $f'$ changes from positive to negative, which happens at $x = 4$. $f$ has a local minimum when $f'$ changes from negative to positive, which happens at $x = 2$ and $x = 6$.

   (c) $f$ is concave upward when $f'$ is increasing, which happens on $[1, 3]$, $[5, 7]$, and $[8, 9]$. $f$ is concave downward when $f'$ is decreasing, which happens on $[0, 1]$, $[3, 5]$, and $[7, 8]$.

   (d) The inflection points of $f$ are the local maxima and minima of $f'$, which happen (from part (b)) at $x = 2$, $x = 4$, and $x = 6$.

7. $f'(x) = 3x^2 - 12 = 3(x^2 - 4)$, so the critical points are $x = 2$ and $x = -2$. For $x < -2$, $f'(x) > 0$. For $-2 < x < 2$, $f'(x) < 0$. Then for $x > 2$, $f'(x) > 0$ again.

   (a) $f$ is increasing on $(-\infty, -2]$ and on $[2, \infty)$. $f$ is decreasing on $[-2, 2]$.

   (b) $f$ has a local maximum at $x = -2$ and a local minimum at $x = 2$.

   (c) $f''(x) = 6x$. For $x < 0$, $f''(x) < 0$, while for $x > 0$, $f''(x) > 0$. Thus $f$ is concave up on $[0, \infty)$ and concave down on $(-\infty, 0]$, and the inflection point occurs at $x = 0$.

12. $f(x) = x^2 e^x$ so $f'(x) = x^2 e^x + 2xe^x = x(x + 2)e^x$. The critical points are $x = 0$ and $x = -2$. For $x < -2$, $f'(x) > 0$. For $-2 < x < 0$, $f'(x) < 0$. For $x > 0$, $f'(x) > 0$.

    (a) $f$ is increasing on $(-\infty, -2]$ and on $[0, \infty)$. $f$ is decreasing on $[-2, 0]$.

    (b) $f$ has a local maximum at $x = -2$. $f$ has a local minimum at $x = 0$.

(c) $f''(x) = x^2e^x + 2xe^x + 2xe^x + 2e^x = (x^2 + 4x + 2)e^x$. We have $f''(x) = 0$ when $x = -2+\sqrt{2}$ and $x = -2-\sqrt{2}$. For $x < -2-\sqrt{2}$, we have $f''(x) > 0$. For $-2 - \sqrt{2} < x < -2 + \sqrt{2}$, we have $f''(x) < 0$. For $x > -2 + \sqrt{2}$, we have $f''(x) > 0$. Thus $f$ is concave up on $(-\infty, -2 - \sqrt{2}]$ and $[-2 + \sqrt{2}, \infty)$, and it is concave down on $[-2-\sqrt{2}, -2+\sqrt{2}]$. The inflection points occur at $x = -2 - \sqrt{2}$ and $x = -2 + \sqrt{2}$.

17. (a) If $f'(2) = 0$ and $f''(2) = -5$, then $f$ has a local maximum at $x = 2$ by the Second Derivative Test.

(b) If $f'(6) = 0$ and $f''(6) = 0$, then we can't say anything about $f$ at $x = 6$. It might have a local maximum or local minimum or stationary point. We'd have to use the First Derivative Test to be sure.