MAT 131 HW solutions (2.8–3.1)

1 Section 2.8

21. If $f(x) = x^3 - 3x + 5$, then $f'(x) = 3x^2 - 3$. The domains of the function and derivative are both $\mathbb{R} = (-\infty, \infty)$ since both functions are polynomials.

24. If $f(x) = \dfrac{3+x}{1-3x}$, then

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{3+x+h}{1-3x-3h} - \frac{3+x}{1-3x}}{h} \\
&= \lim_{h\to 0} \frac{(3+x+h)(1-3x) - (3+x)(1-3x-3h)}{h(1-3x-3h)(1-3x)} \\
&= \lim_{h\to 0} \frac{3+x+h-9x-3x^2-3xh-3+9x+9h-x+3x^2+3xh}{h(1-3x-3h)(1-3x)} \\
&= \lim_{h\to 0} \frac{10h}{h(1-3x-3h)(1-3x)} \\
&= \frac{10}{(1-3x)^2}
\end{aligned}
$$

The function and its derivative both have the same domain: $D = \{x \,|\, x \neq \frac{1}{3}\}$.

31. Not differentiable at $x = -4$ because of a corner; the left and right limits in the derivative definition are different. Not differentiable at $x = 0$ because not continuous.

34. Not differentiable at $x = -1$ because of a discontinuity. Not differentable at $x = 2$ because of a corner.

37. The critical point of a corresponds to the zero of b, so that b is the derivative of a. The critical points of b correspond to the zeroes of c, so that c is the derivative of b. Thus a is f, b is f', and c is f''.

40. The critical points of a do not correspond to zeroes of any other function; therefore the derivative of a is not on the graph. Hence a must represent the highest derivative, which is the jerk. Since the zeroes of a correspond to the critical points of b, a must be the derivative of b, so b is the acceleration. The zero of b is the critical point of c, so c must be the velocity. Finally d, the only graph left, must be the position.

49. (a) The derivative of an even function is an odd function. We know $f(-x) = f(x)$ for all x, and we want to prove that $f'(-x) = -f'(x)$ for all x.

$$\begin{aligned} f'(-x) &= \lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h} && \text{(definition of derivative)} \\ &= \lim_{h\to 0} \frac{f(x-h) - f(x)}{h} && \text{(since } f \text{ is even)} \\ &= \lim_{h\to 0} -\frac{f(x-h) - f(x)}{-h} && \text{(to get `}-h\text{' in denominator)} \\ &= \lim_{t\to 0} -\frac{f(x+t) - f(x)}{t} && \text{(rename } -h = t\text{; as } h \to 0,\ t \to 0) \\ &= -f'(x) && \text{(definition of derivative)} \end{aligned}$$

So $f'(-x) = -f'(x)$ for every x, so f' is odd.

(b) The derivative of an odd function is an even function. We know $f(-x) = -f(x)$ for all x, and we want to prove that $f'(-x) = f'(x)$ for all x.

$$\begin{aligned} f'(-x) &= \lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h} && \text{(definition of derivative)} \\ &= \lim_{h\to 0} \frac{-f(x-h) + f(x)}{h} && \text{(since } f \text{ is odd)} \\ &= \lim_{h\to 0} \frac{f(x-h) - f(x)}{-h} && \text{(to get `}-h\text{' in denominator)} \\ &= \lim_{t\to 0} \frac{f(x+t) - f(x)}{t} && \text{(rename } -h = t\text{; as } h \to 0,\ t \to 0) \\ &= f'(x) && \text{(definition of derivative)} \end{aligned}$$

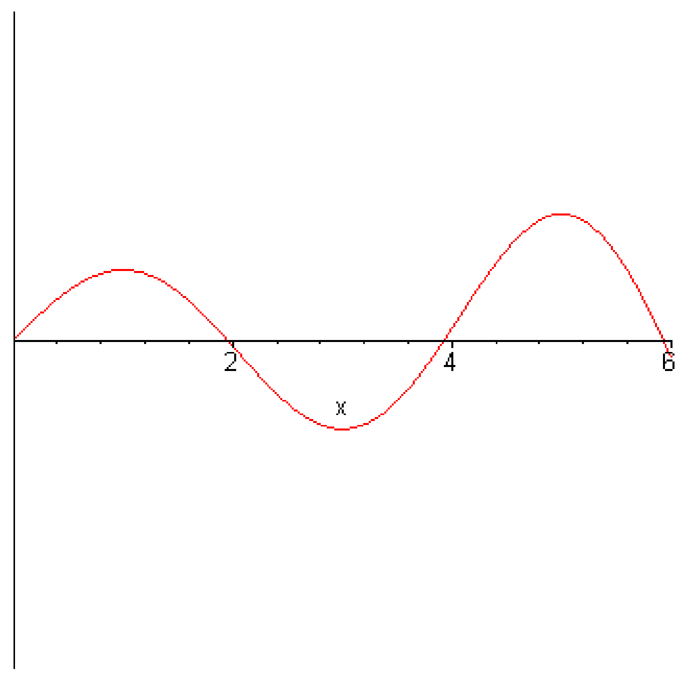
So $f'(-x) = f'(x)$ for every x, so f' is even.

2 Section 2.9

2. (a) f is increasing when f' is positive, on the intervals $(0,1)$ and $(3,5)$. f is decreasing when f' is negative, on the intervals $(1,3)$ and $(5,6)$.

   (b) f has a local maximum when f changes from increasing to decreasing, i.e., at $x = 1$ and $x = 5$. f has a local minimum when f changes from decreasing to increasing, i.e., at $x = 3$.

   (c) Here is a possible graph.



9. Since one learns more material at the beginning of one's studying than at the end (when it's mainly reinforcing previous knowledge), we expect one will learn more in the second hour than in the seventh hour, so that $K(3) - K(2) > K(8) - K(7)$.

   This means that the secant line is steeper from 2 to 3 than from 7 to 8. Since we expect the same kind of thing in general, the secant line slopes should decrease as t increases; therefore also the tangent line slopes should decrease as t increases. So K' is decreasing, and therefore K is concave down.

12. (a) f is increasing for $1 < x < 6$ and for $8 < x < 9$. f is decreasing for $0 < x < 1$ and for $6 < x < 8$.

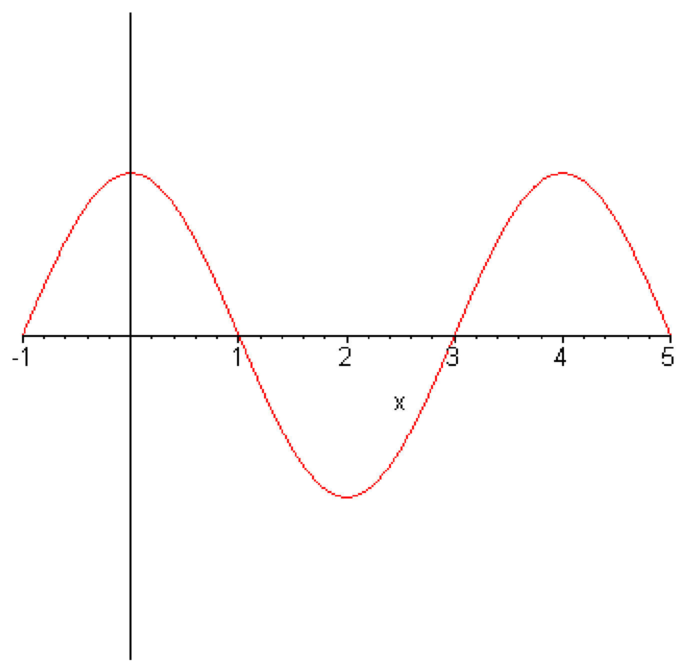
(b) f has a local maximum at $x = 6$. f has local minima at $x = 1$ and $x = 8$.

(c) f is concave up for $0 < x < 2$, for $3 < x < 5$, and for $7 < x < 9$. f is concave down for $2 < x < 3$ and for $5 < x < 7$.

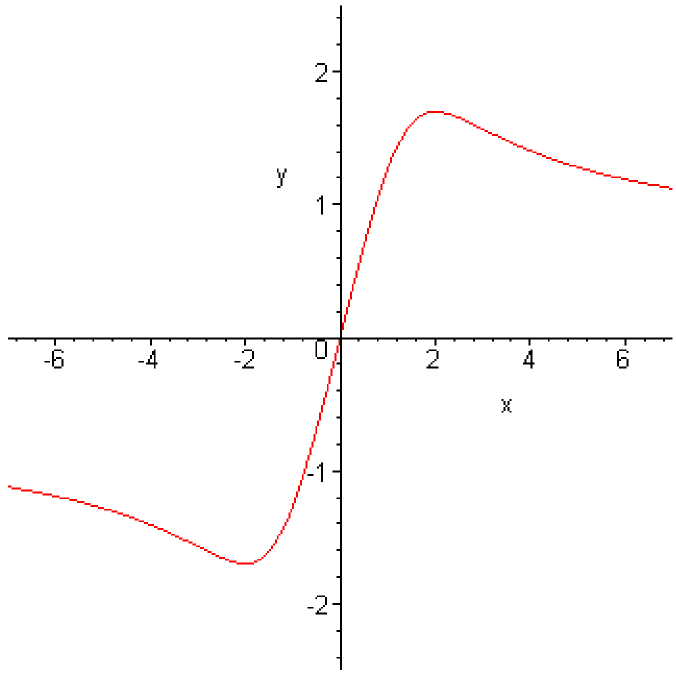
(d) f has inflection points at $x - 2$, $x = 3$, $x = 5$, and $x = 7$.

(e) The graph of f is a bit too hard to draw on a computer, but easy to draw by hand. Ask me in class sometime.

17. Here is a possible graph.



20. Here is a possible graph.

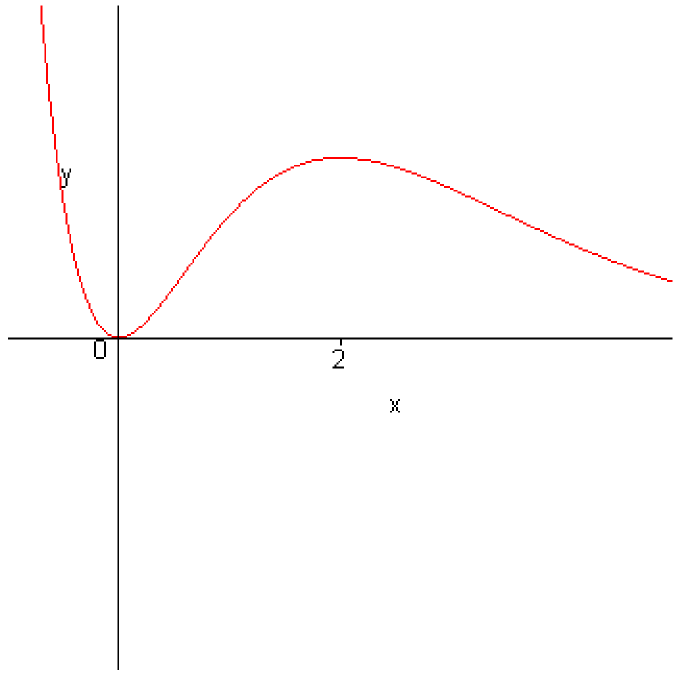


24. (a) $f'(x) = 4x^3 - 4x = 4x(x+1)(x-1)$ and $f''(x) = 12x^2 - 4 = 12(x + \frac{1}{\sqrt{3}})(x - \frac{1}{\sqrt{3}})$.

    (b) f is increasing when $f' > 0$, which occurs on the intervals $-1 < x < 0$ and $x > 1$. f is decreasing when $f' < 0$, which occurs for $x < -1$ and $0 < x < 1$.

    (c) f is concave upward when $f'' > 0$, which occurs for $x > \frac{1}{\sqrt{3}}$ and $x < -\frac{1}{\sqrt{3}}$. f is concave downward when $f'' < 0$, which occurs for $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$.

27. Here is a possible graph.



3 Section 3.1

7. $f'(t) = t^3$

12. $R'(t) = -3t^{-8/5}$

15. $F'(x) = \frac{5}{32}x^4$

24. $\dfrac{du}{dt} = \dfrac{2}{3}t^{-1/3} + \dfrac{2}{3}t^{-2/3}$

25. $\dfrac{dy}{dx} = 4x^3 + 2e^x$, so at $x = 0$, the slope of the tangent line is $m_T = 2$. The slope of the normal line is determined from $m_T m_N = -1$, so $m_N = -\frac{1}{2}$. Thus the tangent line has equation $y - 2 = 2(x - 0)$ and the normal line has equation $y - 2 = -\frac{1}{2}(x - 0)$.

38. $G'(r) = \frac{1}{2}r^{-1/2} + \frac{1}{3}r^{-2/3}$ and $G''(r) = -\frac{1}{4}r^{-3/2} - \frac{2}{9}r^{-5/3}$.

44. $f'(x) = 3x^2 - 8x + 5$ and $f''(x) = 6x - 8 = 6(x - 4/3)$, so f is concave upward for $x > 4/3$.

49. The line $12x - y = 1$ has slope 12, so we are looking for solutions of $y'(x) = 12$. Since $y'(x) = 3x^2$, the solutions of $y'(x) = 12$ are $x = \pm 2$. We see $y(2) = 1 + 2^3 = 9$ and $y(-2) = 1 - 2^3 = -7$, so the equations of the two lines are

$$y - 9 = 12(x - 2) \qquad \text{and} \qquad y + 7 = 12(x + 2).$$