MAT 131 HW solutions (2.5–2.8)

1 Section 2.5

2. (a) The graph can intersect a vertical asymptote, but only if f is explicitly defined (discontinuously) at the asymptote. For example, the function

$$f(x) = \begin{cases} 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

has a vertical asymptote at $x = 0$ but is still defined there.

The graph can intersect a horizontal asymptote; for example,

$$f(x) = \frac{\sin x}{x}$$

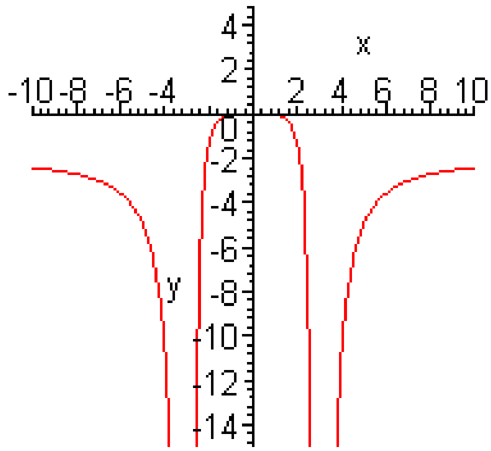
has a horizontal asymptote at $y = 0$ and intersects it at $x = n\pi$ for any nonzero integer n.

(b) A function can have at most two horizontal asymptotes, since any limit is unique; thus there can only be at most one limit as $x \to \infty$ and at most one (possibly different) limit as $x \to -\infty$. An example of a function with two horizontal asymptotes is

$$f(x) = \arctan x.$$

4. a. 2 b. -2 c. $+\infty$ d. $-\infty$ e. $-\infty$
f. Vertical: $x = -2$, $x = 0$, $x = 3$. Horizontal: $y = -2$ and $y = 2$.

10. If f is even, then also $\lim_{x \to -3} f(x) = -\infty$ and $\lim_{x \to -\infty} f(x) = 2$. A possible graph is as follows.



20. $\lim_{x\to\infty} \dfrac{3x+5}{x-4} = 3.$

28. $\lim_{x\to\infty} \dfrac{\sin^2 x}{x^2} = 0$ (by the Squeeze Theorem).

40. A rational function with denominator having factors $(x-1)$ and $(x-3)$ and numerator of the same degree as the denominator will work. For example,

$$f(x) = \frac{x^2+1}{(x-1)(x-3)}$$

2 Section 2.6

8. Compute the derivative:

$$
\begin{aligned}
y'(-1) &= \lim_{h\to 0} \frac{y(-1+h) - y(-1)}{h} \\
&= \lim_{h\to 0} \frac{\big[2(-1+h)^3 - 5(-1+h)\big] - 3}{h} \\
&= \lim_{h\to 0} \frac{[-2 + 6h - 6h^2 + 2h^3 + 5 - 5h] - 3}{h} \\
&= \lim_{h\to 0} \frac{6h - 6h^2 + 2h^3 - 5h}{h} \\
&= \lim_{h\to 0} (1 - 6h + 2h^2) \\
&= 1.
\end{aligned}
$$

Then the tangent line has equation

$$y - 3 = 1(x+1).$$

16. a. Runner A has constant speed through the race. Runner B is slow at the beginning but runs fast at the end to catch up.
    b. The largest vertical distance is about $t = 10$ seconds.
    c. Time $t = 10$ seconds is also when they have the same velocity.

18. a. The velocity in general is

$$
\begin{aligned}
v(a) &= \lim_{h\to 0} \frac{H(a+h) - H(a)}{h} \\
&= \lim_{h\to 0} \frac{58(a+h) - 0.83(a+h)^2 - 58a + 0.83a^2}{h} \\
&= \lim_{h\to 0} (58 - 1.66a - 0.83h) \\
&= 58 - 1.66a
\end{aligned}
$$

    At time $a = 1$, the velocity is $v(1) = 56.34$ meters per second.

    b. See (a).
    c. The arrow hits the moon when $H = 0$, i.e., when $t = 58/0.83 \approx 70$ seconds.

d. The velocity at time $a = 70$ is $v(70) \approx -58$ meters per second.

24. a. (i) 280 thousand per year (ii) 271.5 thousand per year (iii) 311.5 thousand per year.

    b. The instantaneous rate of growth is roughly the average of (ii) and (iii), or 291.5 thousand per year.

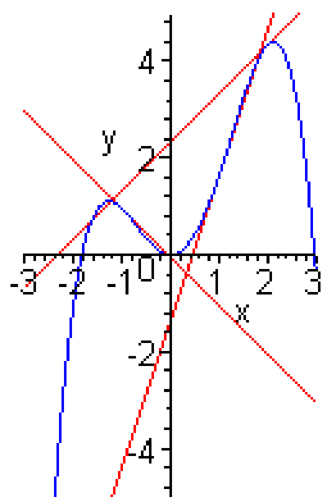
3 Section 2.7

2. In order:

$$0 < \tfrac{1}{2}[f(4) - f(2)] < f(3) - f(2) < f'(2)$$

The rate of change is decreasing, so the tangent at 2 is steeper than the secant from 2 to 3, which is steeper still than the secant from 2 to 4. All of these numbers are positive.

6. A possible solution is graphed below, along with its tangent lines.



10a. $G'(a) = 8a - 3a^2$. We have $G'(2) = 4$ and $G'(3) = -3$, so the tangent line at $(2, 8)$ is $y - 8 = 4(x - 2)$ and that at $(3, 9)$ is $y - 9 = -3(x - 3)$.

16. The derivative is found from

$$\begin{aligned}
f'(a) &= \lim_{x\to a} \frac{f(x)-f(a)}{x-a} \\
&= \lim_{x\to a} \frac{\frac{x^2+1}{x-2} - \frac{a^2+1}{a-2}}{x-a} \\
&= \lim_{x\to a} \frac{1}{x-a} \frac{(x^2+1)(a-2)-(a^2+1)(x-2)}{(x-2)(a-2)} \\
&= \lim_{x\to a} \frac{1}{x-a} \frac{x^2a+a-2x^2-2-a^2x-x+2a^2+2}{(x-2)(a-2)} \\
&= \lim_{x\to a} \frac{1}{x-a} \frac{xa(x-a)-(x-a)-2(x-a)(x+a)}{(x-2)(a-2)} \\
&= \lim_{x\to a} \frac{xa-1-2(x+a)}{(x-2)(a-2)} \\
&= \frac{a^2-1-4a}{(a-2)^2}
\end{aligned}$$

24. Since the limit is as $t \to 1$, the number a must be 1. The function appearing in the numerator is $f(t) = t^4 + t$ minus $f(1) = 2$, so $f(t) = t^4 + t$ will work (or any constant added to it).

4 Section 2.8

2. (The easiest way to do this is sliding the edge of a piece of paper against the graph and looking at intersections.)

   $f'(0) \approx -3, \quad f'(1) \approx 0, \quad f'(2) \approx 1.5, \quad f'(3) \approx 2, \quad f'(4) \approx 0, \quad f'(5) \approx -1.$

3. (a) II; (b) IV; (c) I; (d) III.

10. The function and its derivative are shown below.