MAT 131 HW solutions (5.5)

1 Section 5.5

2. $\int x(4+x^2)^{10}\,dx = \frac{1}{2}\int u^{10}\,du = \frac{1}{22}u^{11} + C = \frac{1}{22}(4+x^2)^{11} + C$

5. $\int \frac{4}{(1+2x)^3}\,dx = 2\int u^{-3}\,du = -u^{-2} + C = -\frac{1}{(1+2x)^2} + C$

11. Let $u = \ln x$. Then

$$\int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$$

14. Let $u = (x^2+1)$. Then

$$\int \frac{x}{(x^2+1)^2}\,dx = \frac{1}{2}\int u^{-2}\,du = -\frac{1}{2}u^{-1} + C = -\frac{1}{2(x^2+1)} + C$$

21. Let $u = \sin\theta$. Then

$$\int \cos\theta \sin^6\theta\,d\theta = \int u^6\,du = \frac{1}{7}u^7 + C = \frac{1}{7}\sin^7\theta + C$$

29. This problem does not require substitution.

$$\int \frac{e^x+1}{e^x}\,dx = \int \left(1 + e^{-x}\right)\,dx = x - e^{-x} + C$$

30. Let $u = e^x + 1$. Then $du = e^x\,dx$. So

$$\int \frac{e^x}{e^x+1}\,dx = \int \frac{1}{u}\,du = \ln u + C = \ln\left(e^x+1\right) + C$$

39. Let $u = (x-1)$. Then

$$\int_0^2 (x-1)^{25}\,dx = \int_{x=0}^{x=2} u^25\,du = \frac{1}{26}u^{26}\Big|_{x=0}^{x=2}$$
$$= \frac{1}{26}(x-1)^{26}\Big|_0^2 = \frac{1}{26}(1^{26} - (-1)^{26}) = 0$$

42. Let $u = x^2$. Then

$$\int x \cos x^2 \, dx = \frac{1}{2} \int \cos u \, du = \frac{1}{2} \sin u + C = \frac{1}{2} \sin x^2 + C,$$

so therefore,

$$\int_0^{\sqrt{\pi}} x \cos x^2 \, dx = \frac{1}{2} \sin x^2 \Big|_0^{\sqrt{\pi}} = \frac{1}{2}(\sin \pi - \sin 0) = 0$$

47. Let $u = x - 1$. Then $du = dx$ and $x = u + 1$, so

$$\int x\sqrt{x-1} \, dx = \int (u+1)\sqrt{u} \, du = \int u^{3/2} + u^{1/2} \, du$$
$$= \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C = \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C,$$

so

$$\int_1^2 x\sqrt{x-1} \, dx = \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2}\Big|_1^2 = \frac{2}{5} + \frac{2}{3} = \frac{16}{15}.$$

50. Let $u = 2x + 1$. Then $du = 2\, dx$ and $x = \frac{u-1}{2}$, so

$$\int \frac{x}{\sqrt{1+2x}} \, dx = \int \frac{\frac{u-1}{2}}{\sqrt{u}} \frac{1}{2} \, du$$
$$= \frac{1}{4} \int u^{1/2} - u^{-1/2} \, du = \frac{1}{6}u^{3/2} - \frac{1}{2}u^{1/2} + C = \frac{1}{6}(2x+1)^{3/2} - \frac{1}{2}(2x+1)^{1/2} + C,$$

and therefore

$$\int_0^4 \frac{x}{\sqrt{1+2x}} \, dx = \left(\frac{1}{6}(2x+1)^{3/2} - \frac{1}{2}(2x+1)^{1/2} \right) \Big|_0^4$$
$$= (\tfrac{1}{6}9^{3/2} - \tfrac{1}{2}9^{1/2}) - (\tfrac{1}{6}1^{3/2} - \tfrac{1}{2}1^{1/2}) = \frac{10}{3}$$

63. Let $u = 2x$. Then $dx = \frac{1}{2}\, du$, and when $x = 0$, $u = 2$, and when $x = 2$, $u = 4$, so

$$\int_0^2 f(2x) \, dx = \int_{u=0}^{u=4} f(u) \, \frac{1}{2} \, du = \frac{1}{2} \int_0^4 f(u) \, du = 5$$

64. Let $u = x^2$. Then $x\,dx = \frac{1}{2}\,du$, and when $x = 0$, $u = 0$, and when $x = 3$, $u = 9$, so

$$\int_0^3 x f(x^2)\,dx = \frac{1}{2}\int_{u=0}^{u=9} f(u)\,du = 2$$