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Appendix

In this appendix, we give the proof of a result that is well–known in the theory of *real* partial differential equations, but somewhat hard to find in the literature for several complex variables. We used this proposition on page 68.

Proposition: *Let* $\alpha : U \to \mathbf{C}$ *be a non–zero holomorphic function that is defined on some open subset of* $\mathbf{C}^2$*, where we denote the complex variables in* $\mathbf{C}^2$ *by* $(u\,,\,v)$*. Furthermore, let* $\phi : U \to \mathbf{C}$ *be a non–constant holomorphic function that is a solution of the linear partial differential equation*

$$\phi_u = \alpha\,\phi_v\ . \tag{1}$$

If $\psi : U \to \mathbf{C}$ *is another solution of this partial differential equation, then for every point* $p_0 = (u_0\,,\,v_0) \in U$ *where* ϕ *has no singularity (i.e.,* $d\phi(p_0) \neq 0$ *), there is a neighborhood* V *of* p_0 *in* U *and a holomorphic function* $h(w)$ *in one complex variable* w *such that, on all of* V*,*

$$\psi = h \circ \phi\ .$$

Proof: Pick a point p_0 where ϕ is non–singular, and assume, without loss of generality, that for all points p in a neighborhood V of p_0, $\phi_v(p) \neq 0$. Let C be any of the values that ϕ takes on V, say $C = \phi(p_1)$, where $p_1 = (u_1\,,\,v_1) \in V$. By the Implicit Mapping Theorem (see [G-R], page 16), there is a holomorphic function $g_C(w)$ with $g_C(u_1) = v_1$ such that

$$\phi(u\,,\,g_C(u)) \equiv C$$

for all u in some neighborhood of u_0 in $\mathbf{C}$. Taking the derivative with respect to u and using (1), we obtain (with $g' := \frac{\partial}{\partial w}\,g$)

$$\begin{aligned} 0 &= \tfrac{\partial}{\partial u}\,\phi(u\,,\,g_C(u)) = \phi_u(u\,,\,g_C(u)) + \phi_v(u\,,\,g_C(u))\,g'_C(u) \\ &= \phi_v(u\,,\,g_C(u))\,[\alpha(u\,,\,g_C(u)) + g'_C(u)]\ . \end{aligned}$$

But we assumed that $\phi_v(p) \neq 0$ for all $p \in V$, and so $g_C(u)$ must satisfy the *ordinary* differential equation

$$g'_C(u) = -\alpha(u\,,\,g_C(u))\ .$$

Note that this differential equation is *completely independent* of ϕ. Thus, if we would repeat the above considerations for our second solution ψ, we would obtain holomorphic functions $\tilde{g}_{\tilde{C}}$ that satisfy the same ordinary differential equation with the same initial condition $\tilde{g}_{\tilde{C}}(u_0) = v_0$. But by the theory of ordinary (complex) differential equations, this means that close to u_0, $g_C(u) = \tilde{g}_{\tilde{C}}(u)$ (see e.g. [Wa], page 110). Therefore, we have shown that the *level curves* of ϕ and ψ must be the same, and thus that (locally) there is a function $h(w)$ such that $\psi = h \circ \phi$; namely, h transforms the values associated with the level curves of ϕ into the values associated with the level curves of ψ.

It remains to show that h is holomorphic. But since $\phi_v(u_1, v_1) \neq 0$, the function $\eta(v) := \phi(u_1, v)$ is invertible in a neighborhood of v_1, and the inverse is holomorphic. Since

$$\psi(u_1, v) = h(\phi(u_1, v)) = h(\eta(v)) ,$$

we have

$$h(w) = \psi(u_1, \eta^{-1}(w)) ,$$

and thus that h is holomorphic.