

Weierstrass Representations of Minimal Real Kähler Submanifolds

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by

Peter Hennes

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Abstract of the Dissertation

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Since the nineteenth century, Weierstrass representations have been used to investigate minimal surfaces in Euclidean 3-space. In the last two decades, it emerged that minimal Kähler submanifolds of Euclidean spaces share many of the features of minimal surfaces. In this dissertation, we try to find similar representations for these minimal real Kähler submanifolds.

First, we modify a method developed by M. Dajczer and D. Gromoll to give a simple way of describing minimal real Kähler hypersurfaces. As an

application, we are able to give local examples of superminimal surfaces in the 4-sphere.

Then, based on the formulae for the classical Weierstrass representation, we find a coordinate system for the homogeneous space of all isotropic complex planes in arbitrary complex vector spaces of dimension at least 5. We utilize this coordinate system to give a local characterization of minimal real Kähler surfaces (of real dimension 4) in Euclidean spaces.

Finally, using this characterization, we are able to give a complete local classification and construction methods for all minimal real Kähler surfaces in Euclidean 6-space, at least away from certain isolated singularities. Employing these construction methods, we also give some explicit new examples for such submanifolds.

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