1. Construct a step-function $f \in L^2(\mathbb{R})$ which is not an element of $L^p(\mathbb{R})$ for any $p \neq 2$.

2. For which $c > 0$ is the function $x \mapsto \frac{1}{\|x\|^c}$ in $L^1(B_n)$, where $B_n = \{x \in \mathbb{R}^n \mid \|x\| < 1\}$?

3. Let X be a random variable on the probability space $(\Omega_1, \mathcal{F}_1, P_1)$ and Y be a random variable on $(\Omega_2, \mathcal{F}_2, P_2)$. Let $\tilde{X}$ and $\tilde{Y}$ be the random variables on $\Omega_1 \times \Omega_2$ induced by X and Y, i.e. $\tilde{X}(x, y) = X(x)$ and $\tilde{Y}(x, y) = Y(y)$. Are $\tilde{X}$ and $\tilde{Y}$ independent?

4. Let f be a Lebesgue measurable function on $[0, 1]$ and denote the graph of f by $\Gamma_f = \{(x, y) \mid f(x) = y\}$. Show that Γ_f is Lebesque measurable and that $m_2(\Gamma_f) = 0$.

5. Let $E \subset [0, 1]$ be the non-measurable set constructed in the book's appendix. Show that the set $E \times \{0\}$ is Lebesgue measurable in $\mathbb{R}^2$, but $E \times [0, 1]$ is not. [Hint: for the second part you may need to show Lebesgue measurable sets in $\mathbb{R}^2$ are translation invariant.]