MAT 511 Fundamental Concepts of Math

Problem Set 5

due Thursday, Oct 16

There is no class on 10/9.

Please prove all your answers. Short and elegant proofs are encouraged but not required.

Problem 1. Some non-intersecting diagonals are selected in a regular n-gon. (A regular n-gon is a polygon with n sides, where all sides are equal and all angles are equal; special cases are an equilateral triangle for $n = 3$, a square for $n = 4$. A diagonal is a straight line connecting two non-adjacent vertices. We are allowed to select several diagonals which come out of the same vertex, as long as no diagonals intersect *inside* the polygon.) Prove that there are always at least two (non-adjacent if $n > 3$) vertices such that no selected diagonal comes from them.

Problem 2. On a $3 \times n$ board there are red, blue, and yellow chips, with n chips of each color, so that each square contains exactly one chip. You are allowed to move the chips around within each of three rows (but not between rows). Prove that you can always move the chips so that each of n columns contains one red, one blue, and one yellow chip.

Problem 3. Find all natural n such that

$$\frac{(2n)!}{(n!)^2} > \frac{4^n}{n+1}$$

and prove your answer.

Please also do questions 8(cipq), 9(df), 10 of §2.4, and 5(a), 6(bc) of §2.5 *Eggen, Smith, St.Andre.*