MAT 341
Midterm II checklist

The exam will focus on the wave equation (sections 3.2,3.3,3.4) and the Laplace equation (aka the potential equation, sections 4.1, 4.2, 4.3, 4.5).

You need to know terminology (wave equation, potential equation, Dirichlet problem, Neumann problem) and appropriate boundary conditions/initial conditions for each situation.

You will still need to know the formulas for the Fourier series of a periodic function f with period $2a$ and know how to use them, as well as sine/cosine Fourier series. Fourier series will not be emphasized (no separate question will be devoted to the Fourier series) but they do come up as solutions to the wave equation and the Laplace equation. You wll not be required to do heavy Fourier series calculations but you may be asked to write the solution as an explicit Fourier series (incorporating the data from the problem) with coefficients given by integrals (without computing the integrals). You may be asked to compute those integrals in *simple cases*. You may also need to find Fourier series for linear combinations of sines and cosines (with matching periods!) such as $3\sin\pi x - 2\cos 7\pi x + \sin 4\pi x$ (this is easy!). There will be no questions about convergence of Fourier series on this exam.

Sturm-Liouville problems will not be specifically emphasized but they do come up when we solve boundary value problems by separation of variables. You should be able to state the relevant eigenvalue problems and find eigenvalues/eigenfunctions in simple examples (see below). You should also be able to state orthogonality of eigenfunctions and to write the solution as a series in eigenfunctions (a generalization of Fourier series).

Questions may include:

Terminology: determine whether the given boundary conditions are Dirichlet-type or Neumann-type; give examples of Dirichlet problem/Neumann problem for the potential equation; write down boundary value/initial value problem for the wave equation (vibratin string). You will not be asked to state the potential equation in polar coordinates (no need to memorize it).

The wave equation:

- Find d'Alambert's solution (as a composition of two waves) for a given vibrating string problem with fixed ends. Find the relevant odd/even extensions for the functions given by the initial conditions. Illustrate the solution graphically, by averaging graphs as in 3.3. You should be able to go through this process with or without prompts at every step.

- Use separation of variables to write the general solution as a series. Use Fourier series to find the coefficients.

- State the eigenvalue problem associated to the given boundary value problem (we get the eigenvalue problem after the separation of variables). In some easy cases, solve the eigenvalue problem (or determine whether, for example, 0 is an eigenvalue, or whether there are positive/negative eignevalues). State orthogonality of eigenfunctions. Write the solution as a series in eigenfunctions.

- For non-homogeneous boundary value problem, write the solution as $u(x,t) = v(x) + w(x,t)$; find the function $v(x)$ and the equation/boundary conditions for $w(x,t)$.

- Describe the general behaviour of the solution; find frequencies of vibration.

The potential equation:

- use separation of variables to solve the Laplace equation with appropriate boundary conditions (Dirichlet-type or Neumann-type as in 4.2, 4.3). You may be asked to solve the question completely (in particular, compute the coefficients of any relevant Fourier series) or just to perform certain steps (for example, find eigenvalues).

- split a given problem into two problems (via $u = u_1 + u_2$) to make separation of variables applicable (you may be asked to do the splitting without solving the resulting problems); explain why the function u given as sum of the two solutions is indeed a solution for the original problem.

- solve the Laplace equation by finding a simple solution $v(x,y)$ that satisfies some of the boundary conditions, and reduce to homogenous boundary conditions on opposite sides (write $u(x,t) = v(x,u) + w(x,t)$ and obtain the equation for w). You may be asked to do the reduction without solving the resulting equation. Similar question for Poisson equation (when a non-homogeneous term is present).

- Laplace equation in a disk: you will not be asked to derive the polar coordinates equation but you must know how to work with it.

- Solve the Dirichlet problem in a disk (or half-disk or quarter-disk, with homogeneous boundary conditions on straight lines). The case at the end of 4.5 is not on the test. You may use solutions of the Cauchy-Euler equation without deriving them. You may be asked to solve the problem completely or just to perform several initial steps (such as separation of variables).
- Find the eigenvalue problem problem associated to a boundary value problem (as above) after the separation of variables; find eigenvalues/eigenfunctions for this problem.

Practice: 3.1 question 3; 3.2 questions 3, 4, 5, 6, 7, 9, 12, 13, 14; 3.3 questions 1-8; p.252-253 questions 1-10; 4.1 questions 1-4, 4.2 questions 5-9; 4.3 questions 1-2, 10; 4.5 questions 1-5; p.299-304 questions 1-9, 11-13, 25. Some of these questions require longer calculations than can be done on exam, but you should know how to approach each problem and be able to fully solve most of them.